




# Studying the Effect of Problem-Based Learning on Mathematics Achievement of Elementary Students

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## How to cite this article:

Ahmad, B., Zulfqar, A., & Khan, A. N. (2024). Studying the Effect of Problem-Based Learning on Mathematics Achievement of Elementary Students. *Journal of Excellence in Social Sciences*, 3(2), 125-136.

Received: 11 March 2024 / Accepted: 7 April 2024 / Published online: 19 May 2024  
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## Abstract

This research aimed to study the impact of problem-based learning on the mathematics achievements of elementary students. Employing a quasi-experimental design with pre and post-tests, the study used convenient sampling to draw its sample from public elementary schools in district Multan. 498 students from 20 public elementary schools participated, with one school undergoing intervention. In this school, the sixth-grade class was divided into two groups: the experimental group (n=25) received problem-based learning instruction, while the control group (n=27) received conventional teaching. A test based on three chapters of the mathematics course, aligned with the lower levels of Bloom's taxonomy, was administered both before and after the intervention. Test scores were analyzed using SPSS, employing descriptive and inferential statistics to derive conclusions. Results indicate a significant improvement in the achievement of the experimental group following the problem-based learning intervention, while only a slight change was observed in the control group. Additionally, the findings suggest that girls outperformed boys in mathematics, contrary to existing research. Limitations and suggestions for future research were also discussed.

**Keywords:** Problem-based Learning; Mathematics Achievement; Quasi-experimental Design; T-test; Elementary Level

## 1 Introduction

In the 21st century, the societal landscape undergoes continual shifts forced by globalization, technological advancements, and demographic changes, opening doors to novel experiences and opportunities. Consequently, there is a pressing need for educational reforms to navigate these

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dynamic changes (Dewanti, et al., 2020). Foremost among the challenges facing education in this era is the imperative to equip students the lifelong learning skills e.g., problem-solving ability, creative thinking, and robust cognitive abilities (Khoiriyah & Husamah, 2018). These competencies not only enable active participation in society but also foster excellence in learning environments and competitiveness in the global arena. Developing such proficiencies demands dedicated time and effort to apply knowledge across diverse contexts and challenges (Carlgren, 2013).

In the current complex world, proficiency in mathematics emerges as a prerequisite for personal advancement (She, et al., 2018). The ability to grasp mathematical concepts, analyze arguments, and discern relationships within them stands as a cornerstone of such proficiency (Mazana, et al., 2019). To fully harness the potential of mathematical principles, effective communication skills and conceptual clarity must be cultivated comprehensively, striving to attain all requisite learning outcomes (Guner, 2020). Mathematics proficiency not only equips students to navigate personal, professional, social, and academic challenges but also empowers them to employ mathematical knowledge in addressing critical issues and devising meaningful solutions (Gure, et al., 2020). However, Mathematics often suffers from a perception as a challenging and uninteresting subject (Almerino et al., 2019).

Evidence suggests that students in Pakistan encounter significant challenges in mathematics performance, often resulting in lower grades (Aslam, et al., 2019; Ashraf, et al., 2022). Weak performance in mathematics contributes to students' disengagement, increased dropout rates, and diminished enthusiasm for learning activities. Moreover, students with lower mathematics achievement could not achieve their goals and leading to decreased motivation (Malik & Rizvi, 2018) and faced stress and anxiety, which can increase dropout rates (Wang et al., 2015).

Therefore, to evaluate the impact of problem-based learning, a learner-centered teaching strategy, on students' mathematics performance, the researcher conducted an experimental study at the elementary level in Pakistan.

### **1.1 Problem Statement**

Evidence indicates that students in Pakistan encounter significant hurdles in mathematics performance, often resulting in lower grades, disengagement, increased dropout rates, and diminished enthusiasm for learning activities. Moreover, students with lower mathematics achievement face obstacles in achieving their goals, leading to decreased motivation and heightened stress and anxiety, which can further exacerbate dropout rates.

Given the identified gaps, it is imperative to assess the influence of problem-based learning on students' mathematics performance. Therefore, this research endeavors to address this gap by conducting an experimental study at the public elementary schools in Pakistan. By exploring the effectiveness of problem-based learning in enhancing mathematics performance, this study aims to provide valuable insights into strategies for improving student engagement, motivation, and achievement in mathematics, thus contributing to the ongoing discourse on educational reform and pedagogical practices.

## **2 Literature Review**

The problem-based learning approach not only enhances students' cognitive processes but also cultivates their ability to intuitively tackle problems and develop critical-thinking skills (Hafiyusholeh, et al., 2015). Continuous exposure to problem-based learning environments fosters the ongoing development of students' mathematical critical thinking abilities, enabling the application of critical thinking in problem-solving scenarios (Padmavathy & Mareesh, 2013).

### **2.1 Philosophical Roots of Problem Based Learning**

Problem-based learning, a pedagogical approach with roots dating back to John Dewey's era, presents students with authentic and meaningful challenges, stimulating inquiry and investigation (Gorghiu, et al., [2015](#); Mann et al., [2021](#)). This method, also termed inquiry-based learning, proves highly effective in nurturing students' foundational skills across various domains. By immersing students in real-world problems, it fosters the construction of knowledge, development of higher-order thinking, and confidence-building (Siagan, et al., [2019](#)). Additionally, problem-based learning serves as an innovative model, particularly beneficial for active and creative learners (Saragih & Habeahan, [2014](#); Gallagher, [2023](#)). By providing an environment conducive to constructing new thoughts relevant to problem-solving, it encourages active engagement and collaboration (Abdullah, et al., [2010](#)).

Problem based learning is deep rooted in constructivism (Phillips, [1997](#)) and learning by doing – progressivism philosophy (Dewey, [1938](#); Almulla, [2020](#)). Constructivism is a philosophy that focuses on how individuals construct knowledge rather than prescribing specific learning methods. Teachers are free to apply constructivist principles in their teaching, but it does not mandate a particular set of pedagogical strategies. Instead, it offers a framework to understand how students learn. Tobin and Tippins ([1993](#)) and Gallagher, ([2023](#)) caution against labeling specific teaching methods as "constructivist," suggesting instead that constructivism be used to analyze instructional approaches and learning processes. Richardson ([2003](#)) emphasizes key aspects of constructivist learning, termed "imperatives," such as student-centered approaches, mixed instruction, group discussions, opportunities for revision, and fostering abstract thinking. These imperatives are especially relevant in problem-based learning environments, where student collaboration and autonomy are crucial. Baden and Major, ([2004](#)) describe various models of team learning in problem-based approaches, emphasizing the role of social constructivism, where students construct knowledge through interaction with teachers and peers.

## 2.2 Conceptualizing Problem Based-Learning

Problem-based learning (PBL) is an educational approach where students collaborate to solve real-world, complex problems, fostering not only content knowledge but also problem-solving, communication, and self-assessment skills. It shifts the traditional teaching paradigm towards a student-centered methodology, where small groups tackle intricate problems, formulating solutions collectively (Misnasanti, et al., [2017](#)). The problem-solving process is influenced by internal factors like prior knowledge and external factors such as the problem's structure and context (Jonassen, [2011](#)). Problems in PBL vary from well-structured to poor-structured, including diagnostic, policy, decision-making, and design issues. Instructional methods in PBL focus on problem-solving skills, with the teacher facilitating rather than instructing directly. The environment in PBL supports diverse problem types, emphasizing learner-centered, reflective learning. Students engage deeply with problems, exploring solutions, making mistakes, and arguing for the best approach (Gagne, [1985](#); Portuguez, [2020](#)). Cognitive support frameworks aid students in constructing robust problem-solving strategies. PBL originated in medical education but has since been adapted to various subjects and grade levels. It emphasizes student-centered learning, nurturing independent, active learners with strong communication and problem-solving skills. PBL promotes meaningful learning by focusing on students' abilities, needs, talents, and interests (Tan & Hung, [2007](#); Goni, [2022](#)). It fosters critical thinking and communication skills, making it particularly suitable for mathematics education. PBL is characterized by active student engagement, where learners progress through problems parallel to the scientific method, developing both content knowledge and problem-solving abilities (Schmidt, et al., [2011](#)). It relies on the dynamic relationship between the learning environment and the learner, with the environment stimulating problems and the learner effectively interpreting and addressing them.

## 2.3 Role of Teacher in Problem-Based Learning

Teachers play a crucial role in enhancing the quality of education through professional development, as noted by Hussain et al. (2021). Their knowledge of high-order thinking skills positively influences student learning, particularly evident in problem-based, project-based, and inquiry-based learning environments (Retnawati et al., 2018). Success in teaching through problem-solving in PBL hinges on support from colleagues and resource partners, as indicated by Hiebert (1999). Rather than solely relying on courses, teachers learn their new roles through teaching and self-reflection (Bransford et al., 2000). While teaching more courses may enhance educators' capacity, it may not necessarily improve students' understanding, as observed by Ball (1993). To fulfill their responsibilities effectively, teachers need opportunities to develop their understanding of mathematical concepts and use them in learning environments, as emphasized by Stein et al. (1999) and Mann (2021). In countries like Japan, novice teachers learn instructional techniques through regular conversations with experienced educators, which are concrete and contextual, focusing on classroom experiences rather than abstract theories (Shimahara & Sakai, 1995). In problem-based learning, teachers have two critical roles: selecting appropriate tasks and organizing classroom discourse to foster students' mathematical understanding.

#### **2.4 Problem-based Curriculum**

As globalization continues to shape education, it becomes essential for schools to equip students with skills that match global standards. In the 21st century, education must shift towards project-based learning, enabling students to tackle real-world problems—a departure from traditional, teacher-centered approaches. This transformation requires a comprehensive overhaul, including curriculum and assessment reforms, and the integration of collaborative techniques (Malik, 2018). Implementing problem-based learning poses various challenges in different curricula. Some advocate for integrating single problem-based modules into specific courses, while others emphasize specific PBL courses at higher grades. However, the most effective implementation involves integrating PBL throughout the entire curriculum, fostering self-regulation skills necessary for navigating complex problems. This approach supports students in adapting to and thriving in problem-based learning environments, particularly as many are not initially adept at solving complex, ill-structured problems (Jonassen, 2011).

#### **2.5 Problem-based Assessment**

In today's classrooms, assessments need to be dynamic, practical, and collaborative, evaluating various aspects of student learning to meet the demands of the modern world (Dewanti et al., 2020). Continuous reforms are necessary to design assessments that can measure complex capabilities effectively. Assessments should progress from measuring basic knowledge to examining problem-solving abilities and mathematical thinking skills, including information gathering and logical reasoning, possibly incorporating technology (Dewanti et al., 2020).

However, assessing 21st-century mathematics skills presents a challenge due to curriculum reforms emphasizing learning experiences (Cheng, et al., 2017). Current assessment methods often fail to gauge students' application of knowledge in new contexts or their ability to use technology for problem-solving, leading to a gap between desired skills and actual proficiency (Dewanti et al., 2020). Assessing mathematical thinking processes, including explanations and diverse strategies, is complex and differs from traditional assessment approaches (Even, 2005; Watson, 2006). Teachers need to adopt innovative assessment practices aligned with learning concepts rather than solely focusing on mathematical content (Duncan & Noonan, 2007). Accurate assessment of 21st century mathematics skills is crucial for evaluating learning and informing future instructional planning.

#### **2.6 Teaching Mathematics in Pakistani Public Schools**

Mathematics poses significant mental challenges, often regarded as the toughest subject in many

countries (Klinek, [2009](#)). Brain-based learning, an effective teaching strategy, is unfortunately underutilized in mathematics classrooms (Johnson, [2003](#); Zamir, [2022](#)). Pakistan faces similar issues, with rote learning prevalent due to a threatening environment and authoritative teaching methods (Ali et al., [2010](#)). Despite efforts, most students in Pakistan fail to meet expected standards in mathematics (Hussain, [2021](#)). Teaching methodology is a crucial factor influencing student achievement in Pakistan, with traditional instructional styles hindering mathematics learning (Farooq & Shah, [2008](#); Khan, [2012](#)). In contrast, developed countries like the United States, Japan, and Australia employ interactive teaching methods that foster strong learning skills (Rehman, [2011](#)). Improving teacher education is vital for enhancing classroom quality and accelerating the learning process (Nafees, [2011](#)). However, traditional teaching methods prevail in Pakistan, resulting in a lack of mathematical development and interest among students (Gul, et al., [2020](#)). In Pakistan, emphasis on memorization over logical thinking contributes to low performance in mathematics. Despite mathematics' academic significance, teaching often prioritizes rote memorization rather than conceptual understanding, leading to poor achievement levels. The challenge lies in fostering contextual understanding and promoting deeper conceptual learning in mathematics education.

### **2.7 Objectives of the Study**

The following objectives were designed to conduct this research:

1. To investigate the impact of problem-based learning (PBL) as a pedagogical approach in enhancing mathematics achievement of students at public elementary school.
2. To analyze the potential difference in mathematics achievement between boys and girls at public elementary school.

### **3 Methodology**

This study employed a quasi-experimental design to assess how problem-based learning affects the mathematics achievement of students enrolled in public elementary schools. The intervention focused on mathematics content from grade six. Pre- and post-intervention data were gathered from a sample of 498 elementary students across ten public schools selected on a convenient basis. Prior to commencing the research, informed consent was obtained from all school principals. Data analysis was conducted using SPSS, encompassing descriptive and inferential statistics to evaluate the impact of intervention on both groups. Ethical protocols were strictly adhered to ensuring the confidentiality and anonymity of all research participants.

#### **3.1 Research Population Sample**

The scope of this study encompassed all public elementary schools in South Punjab. From the 434 public elementary schools located in Multan, a total of 20 schools - ten for boys and ten for girls - were selected using convenience sampling methods (Government of the Punjab, [2023](#)). Within these chosen schools, grade VI was specifically targeted, given their pivotal role as a transitional stage preceding secondary school education. A sample of N=498 students was involved in the research. Furthermore, the intervention was implemented in one school with class six. The class consisted of 52 students. The class was divided into two groups, experimental (25) and control group (27).

#### **3.2 Measures**

To achieve the study objectives, a test was employed as the primary research instrument to gather data from the participants. The test was developed from grade VI to assess their mathematical proficiency. The test served as pre-test to gauge students' mathematical aptitude prior to the intervention, designed around the content of grades VI, test was structured according to the lower three levels of Bloom's taxonomy (Remembering, Understanding, and Applying). The test comprised 50 total marks, with students allotted 75 minutes for completion. The test format remained consistent before and after the intervention, encompassing three question types: Short

answer (Items=05), Fill in the blanks (Items=15), and Multiple-Choice Questions (Items=25).

### 3.3 Design and Implementation of the Intervention

The intervention involved a structured schedule of five lessons per week, each lasting 40 minutes, with one lesson per day. The class and subject teachers collaborated in planning these lessons. Before starting the intervention, students' pre-test scores were analyzed and used to categorize them into high achievers, average achievers, and low achievers. The 27 students in the experimental group were then divided into nine groups, each consisting of three students representing each achievement level. At the outset of the intervention, students were acquainted with the problem-based learning approach in mathematics and briefed on their group allocations. The researcher delivered an introductory lesson on the initial day, outlining that the first three mathematics units would be taught using problem-based learning, with students collaborating in their allotted groups. Over 24 working days, one lesson was conducted each day, with the researcher guiding the intervention using prepared lesson plans. Upon completion of the intervention, a post-test was administered to the experimental group students, administered by the researcher and class teacher. The scores from this post-test were recorded as the students' mathematics achievement following the intervention.

### 3.4 Data Collection

For data collection, the first author of this manuscript obtained consent from school principals. Following approval, the researcher coordinated with class teachers responsible for teaching mathematics to grade VI to schedule the administration of the pre-test. During the designated time slots, the pre-test was administered to students in grades VI with class teachers present. Prior to the test, students were briefed on its purpose, format, duration, and scoring system. They were assured that participation would not affect their individual academic standing within their class or school. The pre-test was conducted across all ten selected schools, involving a total of N=498 students. Subsequently, one school was chosen for the main intervention based on available resources and time constraints. The intervention was implemented with one class, totaling 52 individuals divided into an experimental group (n=25) and a control group (n=27). Each intervention lesson lasted 40 minutes, covering three units of mathematics through problem-based learning over a period of 24 working days. Upon completion of the intervention, the post-test was administered to the experimental group. The test remained the same as conducted before the intervention. Meanwhile, the control group received traditional instruction, and no post-test was conducted, as no alterations were made to the course material or teaching methods. Throughout the research process, strict adherence to research ethics was maintained.

### 3.5 Data Analyses

To assess the research objectives, all tests were marked, and the resulting total scores were recorded in SPSS for analysis. A data sheet was meticulously prepared for this purpose. Initially, descriptive statistics were computed to recognize any notable trends within the dataset. Subsequently, inferential statistics were utilized to evaluate the impact of intervention on the experimental group. Moreover, t-test was employed to gauge the disparity in mathematical achievement between male and female students. Cohen's d was also performed to map out the effect of the intervention and to see the difference between pre and post-test scores. Cohen's d was also performed to map out the effect of the intervention and to see the difference between pre and post-test scores.

## 4 Results

The main purpose of this research was to measure the effect of the problem-based learning intervention on students' mathematics achievement. The following results are inferred after the analyses.

**Table 1:** Comparison of Mathematics Achievement Between pre and post-test in both groups

Variable	Experimental Group		Control Group		t (50)	P	Cohen's d
	M	SD	M	SD			
Pre-test	27.88	6.906	27.63	10.330	0.102	0.919*	0.028
Post-test	35.80	6.403	29.15	9.260	2.989	0.004*	0.835

*N= 52; Students in Experimental Group=25, Students in Control Group=27; \*P>0.05*

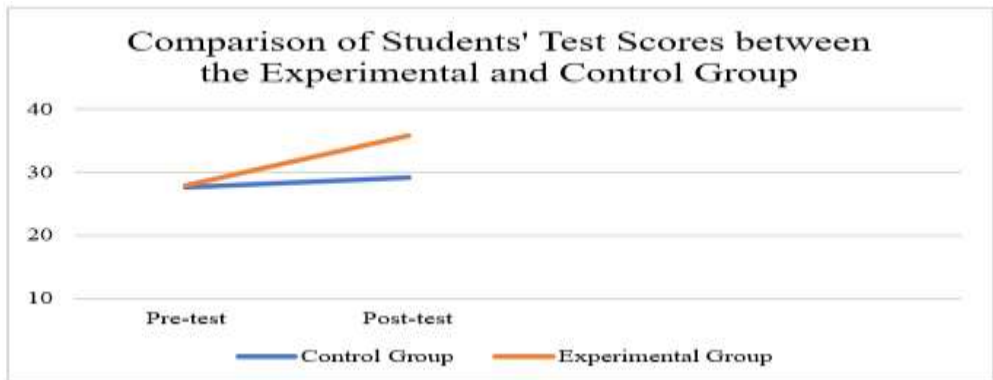
Table 1 indicates that students in both the experimental and control groups performed similarly in the pre-test, with a slight difference in mean scores (M=27.88; SD=6.906) for the experimental group compared to the control group (M=27.63; SD=10.330). However, this disparity in pre-test mean scores between the two groups was not statistically significant, as evidenced by  $t(50) = 0.102$ ,  $*p=0.919 > 0.05$ , and a Cohen's d value of 0.028, falling below the threshold of 0.2. These findings suggest that students in both groups exhibited nearly identical levels of mathematics achievement before the intervention. Furthermore, Table 1 shows that, following the intervention, students in the experimental group scored higher on the post-test (M=35.80; SD=6.403) compared to students in the control group (M=29.15; SD. = 9.260). This difference in mean post-test scores between the two groups was found to be statistically significant, with  $t(50) = 2.989$ ,  $*p=0.004$ , a Cohen's d value of 0.835. This large size effect shows that intervention has a strong impact on the mathematical achievement of students.

**Table 2:** Comparison of Students' Test Scores of Experimental and Control Group

Group	N	Mean of Pre-Test	Mean of Post-Test	Difference
Experimental Group	25	27.88	35.80	7.92
Control Group	27	27.63	29.15	1.52

*N= 52; Students in Experimental Group=25, Students in Control Group=27; \*P>0.05*

Table 2 shows that both groups experimental and control group have an increase in their mean scores from the pre-test to the post-test, experimental group: (M=27.88 to M=35.80); control group: (M=27.63) to (M=29.15). However, the experimental group exhibited a substantially larger increase in scores compared to the control group (Experimental=35.80; Control=29.15). This implies that the intervention implemented with the experimental group likely had a more pronounced effect on improving mathematical achievement compared to the traditional instruction received by the control group. The following figure presents the difference of scores between the experimental and control group.



**Figure 1:** Comparison of Students' Test Scores between the Experimental and Control Group

**Table 3:** Comparison of Mathematics Achievement of Boys and Girls Elementary Students

Girls	Boys
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Variable	M	SD	M	SD	t (496)	P	Cohen's d
Pre-Test	26.38	9.706	23.20	8.906	3.81	0.000*	0.3413

*N* = 498; *Girls* = 247, *Boys* = 251; \**P* < 0.05

Table 3 highlighted that there is a statistically significant difference in pre-test scores between girls and boys in mathematics ( $M_{Girls}=26.38$ ) and ( $M_{Boys}=23.20$ ). Specifically, boys have significantly lower mean pre-test scores compared to girls. The value of Cohen's *d* was  $0.3413 < 1.0$ . The effect size, while not large, is still moderate, indicating a noticeable difference in pre-test scores between the two groups.

## 5 Discussion

Our research findings align with previous studies examining the effects of problem-based learning on mathematics achievement in elementary education. Crowley (2015) and Zamir (2022) study, for instance, investigated the impact of problem-based instruction over an extended period and found that students exposed to this method achieved significantly higher mathematics scores compared to those taught through traditional instruction. Similarly, research by Ndia, et al., (2020) explored the interaction between learning models and multiple intelligences, highlighting that problem-based learning particularly benefited students with spatial intelligence, leading to improved mathematics performance.

Furthermore, a study conducted in Pakistan by Behlol, et al., (2018) supported our findings, demonstrating that implementing a problem-solving approach positively influenced elementary students' mathematics performance across different achievement levels. These studies collectively reinforce the notion that problem-based learning enhances students' mathematics achievement. Moreover, when comparing our findings with those of other studies, it becomes evident that teaching methods significantly impact mathematics performance. Ajai et al. (2013) and Ayub (2021) conducted experimental studies showing that problem-based teaching methods had a lasting impact on students compared to traditional teaching approaches. These results corroborate with our findings, further emphasizing the importance of innovative teaching methods in improving mathematics achievement in public schools. Our findings show that girls perform better in mathematics in contrast to their counterparts, however we could not find such research to corroborate our findings. The existing studies showed that both male and female students are performing equally in mathematics by adopting problem-based learning (Ajai, & Imoko, 2015; Yohannes, et., 2022; Amjad, 2022).

### 5.1 Limitations and Directions for Future Research

The research design was carefully shaped, it is important to acknowledge certain limitations that should be addressed in future research endeavors. Notably, this study exclusively focused on sixth-grade students and implemented the intervention in a single classroom, limiting the generalizability of the problem-based intervention. It is recommended that future research expand its scope by involving multiple schools to assess the effectiveness of such interventions in enhancing elementary students' mathematics achievement. Furthermore, future research efforts could involve collaboration with school principals and teachers to gain insights into the reasons behind the limited adoption of such approaches in schools. In addition to administering tests to students, conducting interviews and quantitative surveys with school administrators and educators could provide valuable perspectives.

### 5.2 Conclusion

Based on the research objectives and findings, it was determined that elementary students in government schools generally have an average level of mathematics achievement, with girls outperforming boys in this subject. The implementation of problem-based learning was found to



enhance both mathematics achievement and cognitive skills among elementary students. Moreover, when comparing two groups of elementary students with similar initial mathematics achievement levels, those taught through problem-based learning demonstrated significant improvements in their mathematics achievement compared to those taught using traditional methods. This highlights a substantial difference in mathematics achievement between students exposed to problem-based learning versus traditional teaching approaches. Additionally, the study concluded that creating a collaborative learning environment, where students actively engage in group discussions, exchange knowledge, and demonstrate sensitivity towards their peers, significantly enhances mathematics achievement. It was observed that when learning is student-centered and the teacher acts as a facilitator or guide, students' mathematics achievement experiences a notable improvement.

## 6 References

- Abdullah, N. I., Tarmizi, R. A., & Abu, R. (2010). The effects of problem-based learning on mathematics performance and affective attributes in learning statistics at form four secondary level. *Procedia-Social and Behavioral Sciences*, 8, 370-376.
- Ajai, J. T., & Imoko, B. I. (2015). Gender differences in mathematics achievement and retention scores: A case of problem-based learning method. *International Journal of research in Education and Science*, 1(1), 45-50.
- Ali, R., Hukamdad, H., Akhter, A., & Khan, A. (2010). Effect of using problem solving method in teaching mathematics. *Asian Social Science*, 6(2), 67-72.
- Almerino Jr, P. M., Etcuban, J. O., De Jose., C. G., & Almerino, J. F. (2019). Students' affective belief as the component in mathematical disposition. *International Electronic Journal of Mathematics Education*, 14(3), 475-487.
- Almulla, M. A. (2020). The effectiveness of the project-based learning (PBL) approach as a way to engage students in learning. *Sage Open*, 10(3), <https://doi.org/10.1177/2158244020938702>
- Amjad, A. I., Habib, M., & Saeed, M. (2022). Effect of brain-based learning on students' mathematics performance at elementary level. *Pakistan Journal of Social Research*, 4(03), 38-51.
- Ashraf, S., Aftab, M. J., Jahan, M., Bahoo, R., & Altaf, S. (2022). Validation of diagnostic Test for students with learning difficulties in mathematics at elementary level. *Multicultural Education*, 8 (5), 157-174.
- Aslam, M., Malik, R., Rawal, S., Rose, P., & Vignoles, A. (2019). Do government schools improve learning for poor students. Evidence from rural Pakistan. *Oxford Review of Education*, 45 (6), 802-824.
- Ayub, A., Gul, R., Malik, M., Sharjeel, M. Y., & Rauf, M. B. (2021). Achievement in mathematics at elementary school level in Quetta city, Balochistan. *Ilkogretim Online-Elementary Education Online*, 20(3), 262-270.
- Baden, M. S., & Major, C. H. 2004. Foundations of problem-based learning. U.K: McGraw-hill education.
- Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, 93(4), 373-393.
- Behlol, M. G., Akbar, R. A., & Sehrish, H. (2018). Effectiveness of problem solving method in teaching mathematics at elementary level. *Bulletin of Education and Research*, 40(1), 231-244.
- Bransford, J. D., Brown, A., & Cocking, R. (2000). How people learn. Washington, DC: National academy press.
- Carlgren, T. (2013). Communication, critical thinking, problem solving: A suggested course for all high school students in the 21st century. *Interchange*, 44(1-2), 63-81.
- Cheng, K. M., Jackson, L., & Lee, W.-o. (2017). Advancing 21st century competencies. *Hong*

*Kong: Asia Society: Center for Global Education.*

- Crowley, B. M. (2015). The effects of problem-based learning on mathematics achievement of elementary students across time. *Masters Theses & Specialist Projects*, 14-46.
- Dewanti, S. S., Kartowagiran, B., & Jailani, R. (2020). Lecturers' experience in assessing 21st - century mathematics competency in Indonesia. *Problems of Education in the 21st Century*, 78(4).
- Dewey, J. (1938). *Education and democracy in the world of today* (1938).
- Duncan, C. R., & Noonan, B. (2007). Factors affecting teachers' grading and assessment practices. *Alberta Journal of Educational Research*, 53(1).
- Even, R. (2005). Using assessment to inform instructional decisions: How hard can it be. *Mathematics Education Research Journal*, 17(3), 45-61.
- Farooq, M. S., & Shah, S. (2008). Students' attitude towards mathematics. *Pakistan Economic and Social Review*, 46(1), 75-83.
- Gagne, R. (1985). *The conditions of learning and theory of instruction* Robert Gagné. New York, NY: Holt, Rinehart ja Winston.
- Gallagher, S. E., & Savage, T. (2023). Challenge-based learning in higher education: An exploratory literature review. *Teaching in Higher Education*, 28(6), 1135-1157. <https://doi.org/10.1080/13562517.2020.1863354>
- Goni, A. M., Tumurang, H., & Ester, K. (2022). Problem based learning (PBL) model and mathematics learning outcomes students. *Specialis Ugdymas*, 1(43), 8277-8284.
- Gorghiu, G., Drăghicescu, L. M., Cristea, S., Petrescu, A.-M., & Gorghiu, L. M. (2015). Problem-based learning-an efficient learning strategy in the science lessons context. *Procedia-social and Behavioral Sciences*, 199, 1865-1870.
- Government of the Punjab, School Education Department. (2011). *School information system*. 2023 (<http://www.sis.punjab.gov.pk>).
- Gul, R., Khan, S. S., & Akhtar, S. (2020). Organizational politics as antecedent of stress in public sector universities of Khyber Pakhtunkhwa. *International Review of Management and Business Research*, 9(2), 150-161.
- Guner, N. (2020). Difficulties encountered by high school students in mathematics. *International Journal of Educational Methodology*, 6(4), 703-713.
- Gure, O. B., Kayri, M., & Erdogan, F. (2020). Analysis of factors effecting PISA 2015 mathematics literacy via educational data mining. *Education & Science/Egitim ve Bilim*, 45, 393-415.
- Hafiyusholeh, M., Hanif, A., & Komaria, R. (2015). Aplikasi metode nilai eigen dalam analytical hierarchy process untuk memilih tempat kerja. *Journal Matematika Mantik*, 1 (1), 6-16.
- Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3-19.
- Hussain, A. A., Akhter, S., Qureshi, A. H., & Khan, K. (2021). Problem-based learning approach for elementary schools: A case study of five years compulsory education system. *Innovation Journal of Social Sciences and Economic Review*, 3(2), 10-14.
- Johnson, T. (2003). *Teaching mathematics with the brain in mind: Learning pure mathematics with meaning and understanding*. Diss. Lethbridge, Alta.: University of Lethbridge, Faculty of Education.
- Jonassen, D. (2011). Supporting problem solving in PBL. *Interdisciplinary Journal of Problem-Based Learning*, 5(2), 95-119.
- Khan, A. W. (2012). Inquiry-based teaching in mathematics classroom in a lower secondary school of Karachi, Pakistan. *International Journal of Academic Research in Progressive Education and Development*, 1(2), 1-7.
- Khoiriyah, A. J., & Husamah, H. (2018). Problem-based learning: Creative thinking skills, problem-solving skills, and learning outcome of seventh grade students. *Journal Pendidikan Biologi Indonesia*, 4(2), 151-160.

- Klinek, S. R. (2009). Brain-based learning: Knowledge, beliefs, and practices of college of education faculty in the Pennsylvania state system of higher education. Diss. *Indiana University of Pennsylvania*.
- Malik, R. H., & Rizvi, A. A. (2018). Effect of classroom learning environment on students' academic achievement in mathematics at secondary level. *Bulletin of Education and Research*, 40 (2), 207-218.
- Malik, R. S. (2018). Educational challenges in 21st century and sustainable development. *Journal of Sustainable Development Education and Research*, 2(1), 9-20.
- Mann, L., Chang, R., Chandrasekaran, S., Coddington, A., Daniel, S., Cook, E., & Smith, T. D. (2021). From problem-based learning to practice-based education: A framework for shaping future engineers. *European Journal of Engineering Education*, 46(1), 27-47. <https://doi.org/10.1080/03043797.2019.1708867>
- Mazana, Y. M., Suero Montero, C., & Olifage, C. R. (2019). Investigating students' attitude towards learning mathematics. *International Electronic Journal of Mathematics Education*, 14(1), 207-231.
- Misnasanti, M., Utami, R., & Suwanto, F. (2017). Problem based learning to improve proportional reasoning of students in mathematics learning. *AIP Conference Proceedings*, 1868(1).
- Nafees, M. (2011). An experimental study on the effectiveness of problem-based versus lecture-based instructional strategies on achievement, retention, and problem-solving capabilities in secondary school general science students. Islamabad: Diss. *International Islamic University Islamabad*.
- Ndia, L., Solihatin, E., & Syahrial, Z. (2020). The effect of learning models and multiple intelligences on mathematics achievement. *International Journal of Instruction*, 13(2), 285-302.
- Orhani, S. (2024). Preparation of tests from the subject of mathematics according to Bloom's taxonomy. *International Journal of Research Publication and Reviews*, 5(2), 2335-2345.
- Padmavathy, R. D., & Mareesh, K. (2013). Effectiveness of problem-based learning in mathematics. *International Multidisciplinary e-Journal*, 2(1), 45-51.
- Phillips, D. (1997). David. Exhibiting authenticity. *Manchester University Press*.
- Portuguez, C. M., & Gomez, Z. M. G. (2020). Challenge based learning: Innovative pedagogy for sustainability through e-learning in higher education. *Sustainability*, 12(10), 4063.
- Rehman, A. U. (2011). Effectiveness of brain-based learning method and conventional method in the teaching of mathematics at secondary level in Pakistan: An experimental study. Diss. *International Islamic University Islamabad*.
- Retnawati, H., Djidu, H., Kartianom, A., & Anazifa, R. D. (2018). Teachers' knowledge about higher-order thinking skills and its learning strategy. *Problems of Education in the 21st Century*, 76(2), 215.
- Richardson, V. (2003). Constructivist pedagogy. *Teachers' college record*, 105(9), 1623-1640.
- Saragih, S., & Habeahan, W. L. (2014). The improving of problem-solving ability and students' creativity mathematical by using problem-based learning in SMP Negeri 2 Siantar. *Journal of Education and Practice*, 5 (35), 123-133.
- Schmidt, H. G., Rotgans, J., & Yew, E. (2011). The process of problem-based learning: what works and why. *Medical Education*, 45(8), 792-806.
- She, H. C., Stacey, K., & Schmidt, W. (2018). Science and mathematics literacy: PISA for better school education. *International Journal of Science and Mathematics Education*, 16, 1-5.
- Shimahara, N. K., & Sakai, A. (1998). 22 Teacher internship and the culture of teaching in Japan. *Education and Training in Japan*, 1(2), 365.
- Siagan, M. V., Saragih, S., & Sinaga, B. (2019). Development of learning materials oriented on problem-based learning model to improve students' problem-solving ability and metacognition ability. *International Electronic Journal of Mathematics Education*, 14 (2), 331-340.

- Stein, M. K., Smith, M., & Silver, E. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. *Harvard Educational Review*, 69(3), 237-270.
- Tan, O. S., & Hung, D. (2007). Problem-Based Learning in e-Learning Breakthroughs. *Singapore: Thomson Learning*.
- Tippins, D. J., Tobin, K. G., & Hook, K. (1993). Ethical decisions at the heart of teaching: Making sense from a constructivist perspective. *Journal of Moral Education*, 22(3), 221-240.
- Wang, H., Yang, C., He, F., Shi, Y., Qu, Q., Rozelle, S., et al. (2015). Mental health and dropout behavior: A cross-sectional study of junior high students in northwest rural China. *International Journal of Educational Development*, 41, 1-12.
- Watson, A. (2006). Some difficulties in informal assessment in mathematics. *Assessment in Education*, 13(3), 289-303.
- Yohannes, Y., Diana, N., & Sukma, Y. (2022). The effect of problem-based learning model toward students' mathematics achievement based on gender. In AIP Conference Proceedings, 2575(1). AIP Publishing.
- Zamir, S., Yang, Z., Wenwu, H., & Sarwar, U. (2022). Assessing the attitude and problem-based learning in mathematics through PLS-SEM modeling. *PloS One*, 17(5), <https://doi.org/10.1371/journal.pone.0266363>