Applications of Mathematical Sciences (2023) 2(2): 15–32
ISSN (E) 2976-7253

Application of n-Rung Orthotriplet Fuzzy Information in Decision Making Problem

Bushra Batool[1]*

How to cite this article:

Batool, B. Application of n-Rung Orthotriplet Fuzzy Information in Decision Making Problem. (2023) *Application of Mathematical Sciences*, 2(2):15-32.

Received: 2 August 2023 / Accepted: 7 October 2023 / Published online: 15 December 2023

Abstract

The n-rung orthotriple fuzzy set (n-ROtFS) is an efficient, generalized and powerful tool for expressing fuzzy information. It can cover more complex imprecise valuation information. Therefore, based on the advantages of n-ROtFSs, this paper presents a new advanced aggregation method to deal with uncertainty in the form of n-ROtFS in real-world problems. We propose Algebraic operational laws based on aggregation operators and their important properties under n-ROtF information. Namely of the propose operators are: n-rung orthotriple weighted average (n-ROtFWA) operator, n-rung orthotriple weighted geometric (n-ROtFWG) operator, n-rung orthotriple ordered weighted average (n-ROtFOWA) operator, n-rung orthotriple ordered weighted geometric (n- ROtFOWG) operator, n-rung orthotriple hybrid average (n-ROtFHA) operator, n-rung orthotriple hybrid geometric (n-ROtFHG) operator. Moreover, we design the algorithm to deal with the problem of uncertainty in decision making problem. Lastly,a digital food security decision making case study is presented to demonstrate the applicability and validity of the proposed methodology.

Keywords: Fuzzy set, orthopair, decision making, operator

1 Introduction

Food security is an important issue in Pakistan that affects a significant portion of the population. The country is faceing various challenges such as climate change, population growth, water scarcity, and poor governance that contribute to nutrition security (Din et al., 2022). The government should take effective measures to address these challenges and ensure that all citizens have access to affordable and wholesome food. This can be achieved through effective policies, investment in agriculture and water management, and good governance (Akhmouch et al., 2021). We established fuzzy decision making methodology to tackle the food security issue in Pakistan.

Pakistan is an agricultural country, and its economy is largely dependent on agriculture. The

[1]Department of Mathematics, University of Sargodha, Sargodha, Pakistan
e-mail: bushra.batool@uos.edu.pk

country produces various crops such as wheat, rice, corn, sugarcane, and cotton. However, despite being an agricultural country, Pakistan faces significant challenges in achieving food security. The country has a large population, and the demand for food increases with population growth (Rehman et al., 2022). However, food production is not to keep up with rising demand, leading to food shortages.

Real-life decision-making problems are very complex and challenging because of the presence of various uncertainties and ambiguities in the information data. Sakr et al. (2023) proposed the fuzzy set theory for efficient modeling of uncertain/ambiguous information. After the pioneering many generalizations and extensions of fuzzy sets (FSs) have been proposed by researchers and are applied in a wide range of application areas. Intuitionistic fuzzy set (IFS), introduced by Mardani et al. (2019), is one of the most notable extensions of the fuzzy set which has been widely studied and implemented in various disciplines. Intuitionistic fuzzy set theory has been successful for the past thirty years designed to solve many problems related to real life situations e.g. decision making (He & Wu, 2019; Peleg & Tu, 2006).

Arya and Kumar (2021), Batool et al. (2022a), and Batool, et al. (2022b) established the Pythagorean fuzzy set (PyFS) as extended form of FFS, with the constraint that the square total of the positive and negative grades of membership is fewer than or equivalent to one. For instance, in a situation where the positive membership value is 0.8 and the negative membership value is 0.3, we can't utilize IFSs because of total of their membership values surpasses one. Consequently, in this circumstance we use PyFSs to bargain the decision-making issues. As a consequence, PyFSs are stronger than IFSs to make a settlement of vagueness in everyday existence issues. After that Batool, et al., (2022a) established the Pythagorean probabilistic hesitant fuzzy set, an extension of Pythagorean fuzzy set in which Batool considered hesitant information along with probabilistic information. (Batool, et al., 2022b) utilize PyPHFS on several real life problems. Then (Smarandache, 2019; Yager, 2016) introduced generalized orthopair fuzzy sets which is an extension of intuitionistic and Pythagorean fuzzy sets.

After that (Guleria & Bajaj, 2020) established the picture fuzzy sets an extended version of Pythagorean fuzzy sets. In Picture fuzzy sets one more membership grade involves. For instance, in a situation when decision makers have more answers: yes, no, refrain and refuse. Voting can be a good example of such a situation, as human voters can be divided into four groups of those who: vote for, abstain, vote against, and refuse to vote. In such situations FS, IFS and PyFS failed to handle uncertainty.

But, there are many complex real life decision making problems exist in which Picture fuzzy set failed to handle uncertainty. To tackle such complex problems Spherical fuzzy set has been established to handle uncertainty more impressively (Ali et al., 2020; Ayyildiz & Taskin, 2022; Jin et al., 2020). Ashraf et al. (2019) utilize spherical fuzzy set on several real life problems. Due to the limited constraints, a novel concept of n rung orthotriplet fuzzy set is established. Therefore, the innovations of this paper are mainly the following aspects: firstly, we utilized novel concept of the n rung orthotriplet fuzzy set (n-ROtFS) to tackle the uncertain information in real life decision making. The motivation of the new concept is that when human opinions have more answer sorts then Pythagorean fuzzy set failed to handle such situation (Akram et al., 2023; Alamoodi et al., 2022). Picture fuzzy sets and Spherical fuzzy sets are special cases of n rung orthotriplet fuzzy set.The n rung orthotriplet fuzzy set ($n-$ ROtFS) is characterized by positive, negative and neutral membership degrees, with the constraint that the nth sum of positive, negative and neutral membership degrees is less than or equal to one. The DMs are limited to a specific domain in PyFS and ignore the neutral degree of membership. Compared to others, neutral membership degree also has some preference. For example, if one DM gives values 0.8, 0.6,0.5

for a positive, negative and neutral membership degrees then in this situation Spherical fuzzy set failed to handle uncertainty since $0.8^2 + 0.6^2 + 0.5^2 = 1.25$ is greater than one but $0.8^3 + 0.6^3 + 0.5^3$ is less than one. Under the proposed concept, the nth level of positive, negative and neutral membership degrees is considered. The domain is extended in case of n rung orthotriplet fuzzy set and when we take $\mathrm{n} = 1$ we obtain Picture fuzzy set and when we take $\mathrm{n} = 2$ we obtain Spherical fuzzy set. More details on the level of difference of opinion of the DMs are provided by the value of positive, negative and neutral membership degrees. The key purpose of this manuscript is to establish n-ROtF model and to select an ideal solution for food security issue in Pakistan.

The motivations of this paper can be established as:

(1) It considers various specialists' conclusions as the neutral membership and breakers them into n-ROtFSs.

(2) In decision analysis, how to choose an ideal solution from among certain alternatives with alike efficacy values to handle food security issue in Pakistan has become serious issue now a days in Pakistan. Generally for government and specialists it is problematic to precisely measure food security in Pakistan. Consequently, this food security issue can be recognized as a MADM issue. This manuscript mainly deliberates an approach to select an appropriate alternative from among certain alternatives to tackle food security issue.

(3) In this paper we utilize n-ROtFS in managing with uncertain data to establish another decision-making algorithm and select an appropriate solution to tackle food security issue in Pakistan.

The arrangement of the manuscript is as per the following. Section 2 gives survey of FSs, IFSs, PyFSs and aggregation operators of n-ROtFSs. In section 3, we presented n-ROtF set and their operational laws. In Section 4, we exhibit the n-ROtF aggregation operators and their properties. In section 5, we exhibit n-ROtF aggregation method to handle vagueness in DMAp. Section 6 explains application of the established MCDM algorithm. In Section 7 conclusion of the manuscript is given.

2 Preliminaries

In this section, we sorts out the essential knowledge about fuzzy sets, hesitant fuzzy sets, probabilistic hesitant fuzzy sets, intuitionistic fuzzy sets and Pythagorean fuzzy sets.

Definition 1. A FS $\mathbb{S}$ in $\mathbb{F}$ is described as

$$\mathbb{S} = \{\langle \wp, \tau_{\mathbb{S}}(\mathbb{S}_{\dot{g}})\rangle \mid \wp \in \mathbb{F}\}$$

for each $\wp \in \mathbb{F}$, the positive membership grade $\tau_{\mathbb{S}}: \mathbb{F} \to \Phi$ specifies the degree to which the element $\wp \in \mathbb{S}$, where $\Phi = [0,1]$.

Definition 2. An IFS $\mathbb{S}$ in $\mathbb{F}$ is described as

$$\mathbb{S} = \{\langle \wp, \tau_{\mathbb{S}}(\mathbb{S}_{\dot{g}}), \exists_{\mathbb{S}}(\wp)\rangle \mid \mathbb{S}_{\dot{g}} \in \mathbb{F}\}$$

for each $\wp \in \mathbb{F}$, the positive membership grade $\tau_{\mathbb{S}}: \mathbb{F} \to \Phi$ and the negative membership grade $\lrcorner_{\mathbb{S}}: \mathbb{F} \to \Phi$ specifies the positive and negative degrees of membership of $\wp$ to the IFS $\mathbb{S}$, respectively, where $\Phi = [0,1]$. Additionally, it is required that $0 \leq \tau_{\mathbb{S}}(\wp) + \exists_{\mathbb{S}}(\wp) \leq 1$

Definition 3. A PyFS $\mathbb{S}$ in $\mathbb{F}$ (Yager & Abbasov, 2013) is described as

$$\mathbb{S} = \{\langle \wp, \tau_{\mathbb{S}}(\mathbb{S}_{\dot{g}}), \exists_{\mathbb{S}}(\wp)\rangle \mid \mathbb{S}_{\dot{g}} \in \mathbb{F}\}$$

for each $\wp \in \mathbb{F}$, the positive membership grade $\tau_{\mathbb{S}}: \mathbb{F} \to \Phi$ and the negative membership grade $\dashv_{\mathbb{S}}: \mathbb{F} \to \Phi$ specifies the positive and negative degrees of membership of $\wp$ to the PyFS $\mathbb{S}$, respectively, where $\Phi = [0,1]$. Additionally, it is required that $0 \leq \tau_{\mathbb{S}}^2(\wp) + \dashv_{\mathbb{S}}^2(\wp) \leq 1$, for each $\wp \in \mathbb{F}$.

Conventionally, $\chi_{\varkappa} = \sqrt{1 - \tau_{\mathbb{S}}^2(\mathbb{S}_{\dot{g}}) - \dashv_{\mathbb{S}}^2(\wp)}$ is said to be degree of hesitancy of $\wp$ to $\mathbb{S}$.

In what follows, we represent by $Py_{\mathsf{F}}\hat{S}(\mathbb{F})$ the group of all Pythagorean fuzzy sets in $\mathbb{F}$. For ease, we will represent the Pythagorean fuzzy number (PyFN) by the pair $\mathbb{S} = (\tau_{\mathbb{S}}, \dashv_{\mathbb{S}})$.

Definition 4. Let $\mathbb{S}_1, \mathbb{S}_2 \in Py_{\mathsf{F}}\hat{S}(\mathbb{F})$. (Yager & Abbasov, 2013), then

(1) $\mathbb{S}_1 \sqsubseteq \mathbb{S}_2$ if and only if $\tau_{\mathbb{S}_1}(\wp) \leq \tau_{\mathbb{S}_2}(\wp)$ and $\exists_{\mathbb{S}_1}(\wp) \geq \exists_{\mathbb{S}_2}(\wp)$ for each $\wp \in \mathbb{F}$. Clearly $\mathbb{S}_1 = \mathbb{S}_2$ if $\mathbb{S}_1 \sqsubseteq \mathbb{S}_2$ and $\mathbb{S}_2 \sqsubseteq \mathbb{S}_1$.

(2) $\mathbb{S}_1 \sqcap \mathbb{S}_2 = \{\min(\tau_{\mathbb{S}_1}(\mathbb{S}_{\dot{g}}), \tau_{\mathbb{S}_2}(\wp)), \max(\dashv_{\mathbb{S}_1}(\wp), \exists_{\mathbb{S}_2}(\wp)) \mid \wp \in \mathbb{F}\}$

(3) $\mathbb{S}_1 \sqcup \mathbb{S}_2 = \{\max(\tau_{\mathbb{S}_1}(\mathbb{S}_{\dot{g}}), \tau_{\mathbb{S}_2}(\wp)), \min(\dashv_{\mathbb{S}_1}(\wp), \dashv_{\mathbb{S}_2}(\wp)) \mid \wp \in \mathbb{F}\}$,

(4) $\mathbb{S}_1^c = \{\exists_{\mathbb{S}_1}(\wp), \tau_{\mathbb{S}_1}(\wp) \mid \wp \in \mathcal{F}\}$.

Definition 5. A HFS $\mathbb{S}$ in $\mathbb{F}$ is (Karamaz & Karaaslan, 2021) described as

$$\mathbb{S} = \{\langle \wp, h_{\varkappa}(\mathbb{S}_{\dot{g}})\rangle \mid \wp \in \mathbb{F}\}$$

where $h_{\varkappa}(\wp)$ is in the form of set, that's contained some possible values in unit interval, i.e., [0,1] which represent the membership degree of $\wp \in \mathbb{F}$ in $\mathbb{S}$.

Definition 6. Let $\mathbb{S}_1, \mathbb{S}_2 \in HFS(\mathbb{F})$ (Karamaz & Karaaslan, 2021), then

(1) $\mathbb{S}_1^c = \bigcup_{\iota \in h_{\mathbb{S}_1}(\mathbb{S}_{\dot{g}})}\{1 - \iota\}$

(2) $(2)\mathbb{S}_1\mathbb{S}_2 = h_{\mathbb{S}_1}(\wp) \barwedge h_{\mathbb{S}_2}(\wp) = \bigcup_{\substack{S_2 \\ \iota_1 \in h_{\mathbb{S}_1}(\wp) \\ \iota_2 \in h_{\mathbb{S}_2}(\wp)}} (\wp)\min\{\iota_1, \iota_2\}$;

(3) $\mathbb{S}_1 \sqcup \mathbb{S}_2 = h_{\mathbb{S}_1}(\wp) \veebar h_{\mathbb{S}_2}(\wp) = \bigcup_{\substack{S_2 \\ \iota_1 \in h_{\mathbb{S}_1}(\wp) \\ \iota_2 \in h_{\mathbb{S}_2}(\wp)}} (\wp)\max\{\iota_1, \iota_2\}$;

(4) $\mathbb{S}_1 \sqcap \mathbb{S}_2 = h_{\mathbb{S}_1}(\wp) \bar{\wedge} h_{\mathbb{S}_2}(\wp) = \bigcup_{\substack{\iota_1 \in h_{\mathbb{S}_1}(\wp) \\ \iota_2 \in h_{\mathbb{S}_2}(\wp)}} S_2(\wp) \min\{\iota_1, \iota_2\};$

Definition 7. A PyHFS $\mathbb{S}$ in $\mathbb{F}$ is (Khan et al., 2017) presented as

$$\mathbb{S} = \{\langle \wp, \tau_{h_{\varkappa}}(\mathbb{S}_{\dot{g}}), \daleth_{h_{\varkappa}}(\wp) \rangle \mid \wp \in \mathbb{F}\}$$

for each $\wp \in \mathbb{F}$, the positive membership grade $\tau_{\mathbb{S}}$ and the negative membership grade $\exists_{\varkappa}$ are sets in some values in [0,1], specifies the possible positive and negative degrees of membership of $\wp$ to the Pythagorean hesitant fuzzy set $\mathbb{S}$, respectively. Furthermore, it is required that $\left(\max(\tau_{h_{\varkappa}}(\wp))\right)^2 + \left(\min\left(\exists_{h_{\varkappa}}(\mathbb{S}_{\dot{g}})\right)\right)^2 \leq 1$ and $\left(\min(\tau_{h_{\varkappa}}(\wp))\right)^2 + \left(\max(\daleth_{h_{\varkappa}}(\wp))\right)^2 \leq 1$. For ease, we will represent the Pythagorean Hesitant Fuzzy Number (PyHFN) by the pair $\mathbb{S} = (\tau_{h_{\varkappa}}, \exists_{h_{\varkappa}})$.

Definition 8. Let $\mathbb{S}_1 = \left(\tau_{h_{\hat{g}_1}}, \daleth_{h_{\hat{g}_1}}\right)$ and $\mathbb{S}_2 = \left(\tau_{h_{\hat{g}_2}}, \daleth_{h_{\hat{g}_2}}\right)$ be two PyHFNs (Khan et al., 2017). The basic operational laws defined as

$$(21)\mathbb{S}_1 \cup \mathbb{S}_2 = \left\{ \bigcup_{\varrho_1 \in \daleth_{h_{\hat{g}_1}}(l_{\dot{g}})\, \varrho_2 \in \daleth_{h_{\hat{g}_2}}(l_{\dot{\xi}})} (\max(\mathbb{S}_1, \mathbb{S}_2)), \bigcup_{\substack{\mathbb{S}_1 \in \tau_{h_{\hat{h}_1}}(l_{\dot{g}}) \\ \mathbb{S}_2 \in \tau_{h_{\hat{g}_2}}(l_{\dot{g}})}}^{\cup} (\min(\varrho_1, \varrho_2)) \right\};$$

$$(2)\mathbb{S}_1 \cap \mathbb{S}_2 = \left\{ \bigcup_{\varrho_1 \in \daleth_{h_{\hat{g}_1}}(l_{\dot{g}})\, \varrho_2 \in \daleth_{h_{\hat{g}_2}}(l_{\dot{\xi}})} (\min(\varrho_1, \varrho_2)), \bigcup_{\substack{\mathbb{S}_1 \in \tau_{h_{\hat{h}_1}}(l_{\dot{g}}) \\ \mathbb{S}_2 \in \tau_{h_{\hat{g}_2}}(l_{\dot{g}})}}^{\cup} (\max(\mathbb{S}_1, \mathbb{S}_2)) \right\};$$

$$(3)\ \mathbb{S}_1^c = \{\exists_{h_{\varkappa}}, \tau_{h_{\varkappa}}\}$$

Definition 9. A PHFS $\mathbb{S}$ in $\mathbb{F}$ is (Xu & Zhou, 2017) described as

$$\mathbb{S} = \{\langle \wp, h_{\varkappa}(\mathbb{S}_{\dot{g}})/\wp_{\dot{g}} \rangle \mid \wp \in \mathbb{F}\}$$

where $h_{\varkappa}(\wp)$ is subset of [0,1] and $h_{\varkappa}(\wp)/\wp_{\dot{g}}$ represent the membership degree of $\wp \in \mathbb{F}$ in $\mathbb{S}$. And $\wp_{\dot{g}}$ represent the possibilities of $h_{\varkappa}(\wp)$, with constraint that $\sum_{\dot{g}} \wp_{\dot{g}} = 1$.

3 n rung Orthotriplet Fuzzy Set

Definition 10. A n rung orthotriplet fuzzy set (n-ROtFS) $\mathbb{S}$ in fixed nonempty set $\mathbb{F}$ is described as

$$\mathbb{S} = \{\wp, R_{\mathbb{S}}(\wp), S_{\mathbb{S}}(\wp), T_{\mathbb{S}}(\wp) \mid \wp \in \mathbb{F}\}$$

Where $R_{\mathbb{S}}(\wp)$ positive, $S_{\mathbb{S}}(\wp)$ neutral and negative $T_{\mathbb{S}}(\wp)$ belongs to $[0,1]$ degrees of membership of $\wp$ to the n rung orthotriplet fuzzy set $\mathbb{S}$, respectively. Additionally, it is required that $0 \leq (R_{\mathbb{S}}(\wp)^n + S_{\mathbb{S}}(\wp)^n + T_{\mathbb{S}}(\wp)^n) \leq 1,$ for $n \geq 1 \forall \wp \in \mathbb{F}$. Also, $\mathfrak{R}_{\mathbb{S}}(\wp) = \sqrt[n]{1 - (R_{\mathbb{S}}(\wp))^n - (S_{\mathbb{S}}(\wp))^n - (T_{\mathbb{S}}(\wp))^n}$ is the degree of hesitancy.

We call the triplet $(R_{\mathbb{S}}, S_{\mathbb{S}}, T_{\mathbb{S}})$ n-ROtFN. n-ROtFS is a generalization of FS, IFS, PyFS, PFS, SFS. The question arise in mind why we need n-ROtFS or what are the limitations in SPS. We consider an example $R_{\mathbb{S}}(\wp) = 0.8, S_{\mathbb{S}}(\wp) = 0.6$ and $T_{\mathbb{S}}(\wp) = 0.5$ which violates the condition that $0 \leq (R_{\mathbb{S}}(\wp)^2 + S_{\mathbb{S}}(\wp)^2 + T_{\mathbb{S}}(\wp)^2) \leq 1$, but $0.8^3 + 0.6^3 + 0.5^3 \leq 1$ and hence the attribute value can be signified by n-ROtFS with $\mathrm{n} = 3$ and so $\mathrm{n} -$ ROtFS can handle such situation very effectively. Therefore n-ROtFS allows the experts to assign degrees by altering the parameter n.

Definition 11. Let $\mathbb{S}_k = \left(R_{\mathbb{S}_k}, S_{\mathbb{S}_k}, T_{\mathbb{S}_k}\right)$ be n-ROtFN ($k = 1,2$). The basic operational laws defined as

(1)$\mathbb{S}_1 \subseteq \mathbb{S}_2$ iff $R_{\mathbb{S}_1} \leq R_{\mathbb{S}_2}, S_{\mathbb{S}_1} \leq S_{\mathbb{S}_2}$ and $T_{\mathbb{S}_1} \geq T_{\mathbb{S}_2}$

(2) $\mathbb{S}_1 = \mathbb{S}_2$ iff $\mathbb{S}_1 \subseteq S_2$ and $\mathbb{S}_2 \subseteq S_1$

(3)$\mathbb{S}_1 \cup \mathbb{S}_2 = \left(\max\left(R_{\mathbb{S}_1}, R_{\mathbb{S}_2}\right), \min\left(S_{\mathbb{S}_1}, S_{\mathbb{S}_2}\right), \min\left(T_{\mathbb{S}_1}, T_{\mathbb{S}_2}\right)\right)$

(4) $\mathbb{S}_1 \cap \mathbb{S}_2 = \left(\min\left(R_{\mathbb{S}_1}, R_{\mathbb{S}_2}\right), \min\left(S_{\mathbb{S}_1}, S_{\mathbb{S}_2}\right), \max\left(T_{\mathbb{S}_1}, T_{\mathbb{S}_2}\right)\right)$

(5)$\mathbb{S}_k^c = \left(T_{\mathbb{S}_k}, S_{\mathbb{S}_k}, R_{\mathbb{S}_k}\right)$

Definition 12. Let $\mathbb{S}_k = \left(R_{\mathbb{S}_k}, S_{\mathbb{S}_k}, T_{\mathbb{S}_k}\right)$ be n-ROtFN ($k = 1,\ 2$) and $\zeta > 0 (\in \mathbb{R})$, then their operations are presented as:

(1) $\mathbb{S}_1 \oplus \mathbb{S}_2 = \left(\sqrt[n]{R_{\mathbb{S}_1}^n + R_{\mathbb{S}_2}^n - R_{\mathbb{S}_1}^n R_{\mathbb{S}_2}^n}, S_{\mathbb{S}_1} \cdot S_{\mathbb{S}_2}, T_{\mathbb{S}_1} \cdot T_{\mathbb{S}_2}\right)$;

(2) $\mathbb{S}_1 \otimes \mathbb{S}_2 = \left(R_{\mathbb{S}_1} \cdot R_{\mathbb{S}_2}, S_{\mathbb{S}_1} \cdot S_{\mathbb{S}_2}, \sqrt[n]{T_{\mathbb{S}_1}^n + T_{\mathbb{S}_2}^n - T_{\mathbb{S}_1}^n \cdot T_{\mathbb{S}_2}^n}\right)$;

(3) $\zeta \cdot \mathbb{S}_1 = \left(\sqrt[n]{1 - \left(1 - R_{\mathbb{S}_1}^n\right)^{\zeta}}, \left(S_{\mathbb{S}_1}\right)^{\zeta}, \left(T_{\mathbb{S}_1}\right)^{\zeta}\right)$

(4)$\mathbb{S}_1^{\zeta} = \left(\left(R_{\mathbb{S}_1}\right)^{\zeta}, \left(S_{\mathbb{S}_1}\right)^{\zeta}, \sqrt[n]{1 - \left(1 - T_{\mathbb{S}_1}^n\right)^{\zeta}}\right)$.

Definition 13. For any $n - ROtFN \mathbb{S} = (R_{\mathbb{S}}, S_{\mathbb{S}}, T_{\mathbb{S}})$ a score function be defined as

$$sc(\mathbb{S}) = \frac{(R_{\mathbb{S}})^n + 1 - (S_{\mathbb{S}})^n + 1 - (T_{\mathbb{S}})^n}{3}$$

Definition 14. For any $n-ROtFN\mathbb{S}=(R_{\mathbb{S}},S_{\mathbb{S}},T_{\mathbb{S}})$ an accuracy function is defined as

$$ac(\mathbb{S})=(R_{\mathbb{S}})^{n}-(T_{\mathbb{S}})^{n}$$

Definition 15. Let $\mathbb{S}_k=\left(R_{\mathbb{S}_k},S_{\mathbb{S}_k},T_{\mathbb{S}_k}\right)$ be n-ROtFN ($k=1,2$) Then by using above definition, comparison of n-ROtFNs can be described as

(1) If $sc(\mathbb{S}_1)>sc(\mathbb{S}_2)$, then $\mathbb{S}_1>\mathbb{S}_2$.

(2) If $sc(\mathbb{S}_1)=sc(\mathbb{S}_2)$, and $ac(\mathbb{S}_1)>ac(\mathbb{S}_2)$ then $\mathbb{S}_1>\mathbb{S}_2$

4 Aggregation Information FOR N-ROtFNs

This section presents some aggregation operators for n rung orthotriplet fuzzy numbers derived from operational laws.

Definition 16. Let $\mathbb{S}_{\jmath}=\left\{\wp,R_{\mathbb{S}_j}(\wp),S_{\mathbb{S}_j}(\wp),T_{\mathbb{S}_j}(\wp)\mid\wp\in\mathbb{F}\right\}(\jmath=1,2,\dots\dots,d)$ be any group of n-ROtFNs and $n-ROtFWA\colon n-ROtFN^{d}\longrightarrow n-ROtFN$. Then n-ROtFWA operator can be described as

$$n-\text{ROtFW }A(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_d)=\beth_1\mathbb{S}_1\oplus\beth_2\mathbb{S}_2\oplus\dots.\oplus\beth_d\mathbb{S}_d$$

where $\beth=(\beth_1,\beth_2,\dots,\beth_d)^{T}$ are the weights of $\mathbb{S}_{\jmath}\in[0,1]$ with $\sum_{\jmath=1}^{d}\beth_{\jmath}=1$.

Theorem 1. Let $\mathbb{S}_{\jmath}=\left\{\wp,R_{\mathbb{S}_j}(\wp),S_{\mathbb{S}_j}(\wp),T_{\mathbb{S}_j}(\wp)\mid\wp\in\mathbb{F}\right\}(\jmath=1,2,\dots.,d)$ be any group of n-ROtFNs. Then the aggregation result using $n-ROtFWA$, we can achieve the following

$$n-\text{ROtFWA }A(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_d)=\begin{pmatrix}\sqrt[n]{1-\Pi_{\jmath=1}^{d}\left(1-\left(R_{\mathbb{S}_{\jmath}}\right)^{n}\right)^{\beth_{\jmath}}},\Pi_{\jmath=1}^{d}\left(S_{\mathbb{S}_{\jmath}}\right)^{\beth_{\jmath}},\\ ,\Pi_{\jmath=1}^{d}\left(T_{\mathbb{S}_{\jmath}}\right)^{\beth_{\jmath}}\end{pmatrix}$$

where $\beth=(\beth_1,\beth_2,\dots.,\beth_d)^{T}$ are the weights of $\mathbb{S}_{\jmath}\in[0,1]$ with $\sum_{\jmath=1}^{d}\beth_{\jmath}=1$.

Proof. We will demonstrate the theorem by following the steps mathematical induction on d. Since for each $\jmath,\mathbb{S}_{\jmath}=\left\{\wp,R_{\mathbb{S}_j}(\wp),S_{\mathbb{S}_j}(\wp),T_{\mathbb{S}_j}(\wp)\mid\wp\in\mathbb{F}\right\}$ Where $R_{\mathbb{S}}(\wp),S_{\mathbb{S}}(\wp)$ and $T_{\mathbb{S}}(\wp)$ belongs to $[0,1]$ and $(R_{\mathbb{S}}(\wp)^{n}+S_{\mathbb{S}}(\wp)^{n}+T_{\mathbb{S}}(\wp)^{n})\leq 1$, for $n\geq 1$

Step 1: When $d=2$, we have $n-\text{ROtFW }A(\mathbb{S}_1,\mathbb{S}_2)=\beth_1\mathbb{S}_1\oplus\beth_2\mathbb{S}_2$ Thus, by the operation of n-ROtFEs, we achieve

$$\beth_1\cdot\mathbb{S}_1=\left(\sqrt[n]{1-\left(1-R_{\mathbb{S}_1}^{n}\right)^{\beth_1}},\left(S_{\mathbb{S}_1}\right)^{\beth_1},\left(T_{\mathbb{S}_1}\right)^{I_1}\right)$$

$$\boldsymbol{I}_2\cdot\mathbb{S}_2=\left(\sqrt[n]{1-\left(1-R_{\mathbb{S}_2}^{n}\right)^{\beth_2}},\left(S_{\mathbb{S}_2}\right)^{\beth_2},\left(T_{\mathbb{S}_2}\right)^{I_2}\right)$$

Then

$$n-\text{ROtFWA}\,(\mathbb{S}_1,\mathbb{S}_2)=ℶ_1\mathbb{S}_1\oplus ℶ_2\mathbb{S}_2$$

$$=\left(\left(\sqrt{\begin{array}{c}1-\left(1-R_{\mathbb{S}_1}^{n}\right)^{ℶ_1}+\left(1-(1-R_{\mathbb{S}_2}^{n})^{ℶ_2}\right)\\-\left(1-(1-R_{\mathbb{S}_1}^{n})^{ℶ_1}\right).\left(1-(1-R_{\mathbb{S}_2}^{n})^{ℶ_2}\right)\end{array}}\right),\atop (S_{\mathbb{S}_1})^{ℶ_1}.(S_{\mathbb{S}_2})^{ℶ_2},(T_{\mathbb{S}_1})^{ℶ_1}.(T_{\mathbb{S}_2})^{ℶ_2}\right)$$

$$=\left\{\left(\sqrt{1-\left(1-R_{\mathbb{S}_1}^{n}\right)^{ℶ_1}\left(1-R_{\mathbb{S}_2}^{n}\right)^{ℶ_2}}\right),\left(S_{\mathbb{S}_1}\right)^{ℶ_1}\cdot\left(S_{\mathbb{S}_2}\right)^{ℶ_2},\left(T_{\mathbb{S}_1}\right)^{ℶ_1}\cdot\left(T_{\mathbb{S}_2}\right)^{ℶ_2}\right\}$$

$$=\left\{\sqrt{1-\Pi_{\hat{\jmath}=1}^{2}\left(1-R_{\mathbb{S}_{\hat{\jmath}}}^{n}\right)^{I_{\hat{\jmath}}}},\Pi_{\hat{\jmath}=1}^{2}\left(S_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}},\Pi_{\hat{\jmath}=1}^{2}\left(T_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}}\right\}$$

Thus, the result holds for $d=2$.

Step 2: Assume that the result holds for $d=z$, we have

$$n-\text{ROtFWA}\,(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_z)=\left(\begin{array}{c}\sqrt{1-\Pi_{\hat{\jmath}=1}^{z}\left(1-R_{\mathbb{S}_{\hat{\jmath}}}^{n}\right)^{ℶ_{\hat{\jmath}}}}\\\Pi_{\hat{\jmath}=1}^{z}\left(S_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}},\Pi_{\hat{\jmath}=1}^{z}\left(T_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}}\end{array}\right)$$

Step 3: When $d=z+1$, then we have

$$n-\text{ROtFW }A(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_{z+1})=\bigoplus_{\hat{\jmath}=1}^{z}ℶ_{\hat{\jmath}}\mathbb{S}_{\hat{\jmath}}\oplus ℶ_{z+1}\mathbb{S}_{z+1}$$

$$=\left(\begin{array}{c}\sqrt{1-\Pi_{\hat{\jmath}=1}^{z}\left(1-R_{\mathbb{S}_{\hat{\jmath}}}^{n}\right)^{ℶ_{\hat{\jmath}}}},\\\Pi_{\hat{\jmath}=1}^{z}\left(S_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}},\Pi_{\hat{\jmath}=1}^{z}\left(T_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}}\end{array}\right)$$

$$\oplus\left(\begin{array}{c}\sqrt{1-\left(1-R_{\mathbb{S}_{z+1}}^{n}\right)^{ℶ_{z+1}}}\\\left(S_{\mathbb{S}_{z+1}}\right)^{ℶ_{z+1}},\left(T_{\mathbb{S}_{z+1}}\right)^{ℶ_{z+1}}\end{array}\right)$$

$$=\left(\begin{array}{c}\sqrt{\begin{array}{c}1-\Pi_{\hat{\jmath}=1}^{z}\left(1-R_{\mathbb{S}_{\hat{\jmath}}}^{n}\right)^{ℶ_{\hat{\jmath}}}+\left(1-\left(1-R_{\mathbb{S}_{z+1}}^{n}\right)^{ℶ_{z+1}}\right)\\-\left(1-\Pi_{\hat{\jmath}=1}^{z}\left(1-R_{\mathbb{S}_{\hat{\jmath}}}^{n}\right)^{ℶ_{\hat{\jmath}}}\right)\left(1-\left(1-R_{\mathbb{S}_{z+1}}^{n}\right)^{ℶ_{z+1}}\right)\end{array}},\\\Pi_{\hat{\jmath}=1}^{z}\left(S_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}}\left(S_{\mathbb{S}_{z+1}}\right)^{ℶ_{z+1}},\Pi_{\hat{\jmath}=1}^{z}\left(T_{\mathbb{S}_{\hat{\jmath}}}\right)^{ℶ_{\hat{\jmath}}}\left(T_{\mathbb{S}_{z+1}}\right)^{ℶ_{z+1}}\end{array}\right)$$

$$= \begin{pmatrix} \sqrt{1-\Pi_{\hat{j}=1}^{z}\left(1-R_{\mathbb{S}_{\hat{j}}}^{n}\right)^{\beth_{\hat{j}}}\left(1-R_{\mathbb{S}_{z+1}}^{n}\right)^{\beth_{z+1}}}, \\ \Pi_{\hat{j}=1}^{z}\left(S_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}}\left(S_{\mathbb{S}_{z+1}}\right)^{\beth_{z+1}}, \Pi_{\hat{j}=1}^{z}\left(T_{\mathbb{S}_{\hat{j}}}^{\beth_{\hat{j}}}\left(T_{\mathbb{S}_{z+1}}\right)^{\beth_{z+1}}\right) \end{pmatrix}$$

$$= \begin{pmatrix} \sqrt{1-\Pi_{\hat{j}=1}^{z+1}\left(1-R_{\mathbb{S}_{\hat{j}}}^{n}\right)^{\beth_{\hat{j}}}}, \Pi_{\hat{j}=1}^{z+1}\left(S_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}}, \\ \Pi_{\hat{j}=1}^{z+1}\left(T_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}} \end{pmatrix}$$

Thus

$$n - \text{ROtFWA } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \begin{pmatrix} \sqrt{1-\Pi_{\hat{j}=1}^{d}\left(1-R_{\mathbb{S}_{\hat{j}}}^{n}\right)^{\beth_{\hat{j}}}}, \Pi_{\hat{j}=1}^{d}\left(S_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}}, \\ \Pi_{\hat{j}=1}^{d}\left(T_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}} \end{pmatrix}$$

Definition 17. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs and $n -$ ROtFWG : $n - ROtFN^d \longrightarrow n - ROtFN$. Then n-ROtFWG operator can be described as

$$n - ROtFWG(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = I_1^{\mathbb{S}_1} \oplus \mathfrak{I}_2^{\mathbb{S}_2} \oplus \dots . \oplus \boldsymbol{I}_d^{\mathbb{S}_d}$$

where $\beth = (\beth_1, \beth_2, \dots ., \beth_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \beth_{\hat{j}} = 1$.

Theorem 2. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_j}(\wp), S_{\mathbb{S}_j}(\wp), T_{\mathbb{S}_J}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots ., d)$ be any group of n-ROtFNs. Then the aggregation result using $n - ROtFWG$, we can achieve the following

$$n - \text{ROtFWG } (\mathbb{S}_1, \mathbb{S}_2, \dots ., \mathbb{S}_d) = \begin{pmatrix} \Pi_{\hat{j}=1}^{d}\left(T_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}}, \Pi_{\hat{j}=1}^{d}\left(S_{\mathbb{S}_{\hat{j}}}\right)^{\beth_{\hat{j}}}, \\ \sqrt[n]{1-\Pi_{\hat{j}=1}^{d}\left(1-\left(R_{\mathbb{S}_{\hat{j}}}\right)^{n}\right)^{\beth_{\hat{j}}}} \end{pmatrix}$$

where $\beth = (\beth_1, \beth_2, \dots, \beth_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \beth_{\hat{j}} = 1$.

Definition 18. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs and $n - ROtFOWA: n - ROtFN^d \longrightarrow n - ROtFN$. Then n-ROtFOWA operator can be described as

$$n - \text{ROtFOW } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \beth_1 \mathbb{S}_{\eta(1)} \oplus \beth_2 \mathbb{S}_{\eta(2)} \oplus \dots . \oplus \beth_d \mathbb{S}_{\eta(d)}$$

where $\eta(\hat{j})$ is denoted for ordered and $(\eta(1), \eta(2), \dots ., \eta(d))$ is a permutation of $(1,2,3, \dots, d)$, subject to $\mathbb{S}_{\eta(\hat{j}-1)} \geq \mathbb{S}_{\eta(\hat{j})}$ for all $\hat{j}$. Also $\beth = (\boldsymbol{I}_1, \beth_2, \dots, \boldsymbol{J}_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \rfloor_{\hat{j}} = 1$.

Theorem 3. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs. Then the aggregation result using $n - ROtFOWA$, we can achieve the following

$$n - \text{ROtFOW } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \begin{pmatrix} \sqrt[n]{1 - \Pi_{\hat{j}=1}^{d}\left(1 - \left(R_{\mathbb{S}_{\eta(\hat{j})}}\right)^{n}\right)^{\Xi_{\hat{j}}}}, \Pi_{\hat{j}=1}^{d}\left(S_{\mathbb{S}_{\eta(\hat{j})}}\right)^{\boldsymbol{I}_{\hat{j}}} \\ ,, \Pi_{\hat{j}=1}^{d}\left(T_{\mathbb{S}_{\eta(\hat{j})}}\right)^{\beth_{\hat{j}}} \end{pmatrix}$$

where $\beth = (\boldsymbol{I}_1, \beth_2, \dots, \boldsymbol{J}_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \boldsymbol{J}_{\hat{j}} = 1$.

Definition 19. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs and $n - ROtFOWG: n - ROtFN^d \longrightarrow n - ROtFN$. Then n-ROtFOWG operator can be described as

$$n - \text{ROtFOWG } (\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \left(\mathbb{S}_{\eta(1)}\right)^{\beth_1} \oplus \left(\mathbb{S}_{\eta(2)}\right)^{\beth_2} \oplus \dots .. \oplus \left(\mathbb{S}_{\eta(d)}\right)^{\beth_d}$$

where $\eta(\hat{j})$ is denoted for ordered and $(\eta(1), \eta(2), \dots ., \eta(d))$ is a permutation of $(1,2,3, \dots, d)$, subject to $\mathbb{S}_{\eta(\hat{j}-1)} \geq \mathbb{S}_{\eta(\hat{j})}$ for all $\hat{j}$. Also $\beth = (\beth_1, \boldsymbol{I}_2, \dots, \beth_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \beth_{\hat{j}} = 1$.

Theorem 4. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs. Then the aggregation result using $n - ROtFOWG$, we can achieve the following

$$n - \text{ROtFOWG } (\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \begin{pmatrix} \prod_{\hat{j}=1}^{d} \left(T_{\mathbb{S}_{\eta(\hat{j})}}\right)^{\beth_{\hat{j}}}, \Pi_{\hat{j}=1}^{d}\left(S_{\mathbb{S}_{\eta(\hat{j})}}\right)^{\beth_{\hat{j}}} \\ \sqrt[n]{1 - \Pi_{\hat{j}=1}^{d}\left(1 - \left(R_{\mathbb{S}_{\eta(\hat{j}}}\right)^{n}\right)^{\beth_{\hat{j}}}} \end{pmatrix}$$

where $\beth = (\beth_1, \beth_2, \dots, \beth_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \beth_{\hat{j}} = 1$.

Definition 20. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs and $n - ROtFHWA: n - ROtFN^d \longrightarrow n - ROtFN$. Then n-ROtFHWA operator can be described as

$$n - \text{ROtFHW } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \omega_1 \dot{S}_{\eta(1)} \oplus \omega_2 \mathfrak{S}_{\eta(2)} \oplus \dots . \oplus \omega_d \mathcal{S}_{\eta(d)}$$

where $\beth = (\beth_1, \beth_2, \dots, \beth_d)^T$ are the weights of $\mathbb{S}_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \beth_{\hat{j}} = 1$ and gth biggest weighted value is $\mathbb{S}_{\eta(\hat{j})}\left(\mathbb{S}_{\eta(\hat{j})} = \eta \boldsymbol{J}_{\hat{j}} \mathbb{S}_{\eta(\hat{j})}\right)$. Also the associated weighted vector $\omega_{\hat{j}}$ with $\omega_{\hat{j}} \in [0,1]$ with $\sum_{\hat{j}=1}^{d} \omega_{\hat{j}} = 1$

Theorem 5. Let $\mathbb{S}_{\hat{j}} = \left\{\wp, R_{\mathbb{S}_{\hat{j}}}(\wp), S_{\mathbb{S}_{\hat{j}}}(\wp), T_{\mathbb{S}_{\hat{j}}}(\wp) \mid \wp \in \mathbb{F}\right\} (\hat{j} = 1,2, \dots \dots, d)$ be any group of n-ROtFNs. Then the aggregation result using $n - ROtFHWA$, we can achieve the following

$$n-\text{ROtFHW } A(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_d)=\begin{pmatrix}\sqrt[n]{1-\Pi_{\jmath=1}^{d}\left(1-\left(R_{\mathbb{S}_{\eta(\jmath)}}\right)^{n}\right)^{ℶ_\jmath}},\Pi_{\jmath=1}^{d}\left(S_{\hat{\mathbb{S}}_{\eta(\jmath)}}\right)^{ℶ_\jmath}\\ ,,\Pi_{\jmath=1}^{d}\left(T_{\mathbb{S}_{\eta(\jmath)}}\right)^{ℶ_\jmath}\end{pmatrix}$$

where $ℶ=(\boldsymbol{I}_1,ℶ_2,\dots,\boldsymbol{J}_d)^T$ are the weights of $\mathbb{S}_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}ℶ_\jmath=1$ and gth biggest weighted value is $\mathbb{S}_{\eta(\jmath)}\left(\mathbb{S}_{\eta(\jmath)}=\eta ℶ_\jmath\mathbb{S}_{\eta(\jmath)}\right)$. Also the associated weighted vector $\omega_\jmath$ with $\omega_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}\omega_\jmath=1$

Definition 21. Let $\mathbb{S}_\jmath=\left\{\wp,R_{\mathbb{S}_\jmath}(\wp),S_{\mathbb{S}_\jmath}(\wp),T_{\mathbb{S}_\jmath}(\wp)\mid\wp\in\mathbb{F}\right\}(\hat{\jmath}=1,2,\dots\dots,d)$ be any group of n-ROtFNs and $n-ROtFHWG: n-ROtFN^d\longrightarrow n-ROtFN$. Then n-ROtFHWG operator can be described as

$$n-\text{ROtFHWG }(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_d)=\mathfrak{S}_{\eta(1)}^{\omega_1}\oplus\mathbb{S}_{\eta(2)}^{\omega_2}\oplus\dots..\oplus\mathbb{S}_{\eta(d)}^{\omega_d}$$

where $ℶ=(ℶ_1,ℶ_2,\dots,\boldsymbol{I}_d)^T$ are the weights of $\mathbb{S}_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}ℶ_\jmath=1$ and gth biggest weighted value is $\mathbb{S}_{\eta(\jmath)}\left(\mathbb{S}_{\eta(\jmath)}=\eta\boldsymbol{J}_\jmath\mathbb{S}_{\eta(\jmath)}\right)$. Also the associated weighted vector $\omega_\jmath$ with $\omega_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}\omega_\jmath=1$

Theorem 6. Let $\mathbb{S}_\jmath=\left\{\wp,R_{\mathbb{S}_\jmath}(\wp),S_{\mathbb{S}_\jmath}(\wp),T_{\mathbb{S}_\jmath}(\wp)\mid\wp\in\mathbb{F}\right\}(\hat{\jmath}=1,2,\dots\dots,d)$ be any group of n-ROtFNs. Then the aggregation result using $n-ROtFHWG$, we can achieve the following

$$n-\text{ROtFHWG }(\mathbb{S}_1,\mathbb{S}_2,\dots,\mathbb{S}_d)=\begin{pmatrix}\Pi_{\jmath=1}^{d}\left(T_{\hat{S}_{\eta(\jmath)}}\right)^{ℶ_\jmath},\Pi_{\jmath=1}^{d}\left(S_{\hat{S}_{\eta(\jmath)}}\right)^{ℶ_\jmath}\\ \sqrt[n]{1-\Pi_{\jmath=1}^{d}\left(1-\left(R_{\hat{\mathbb{S}}_{\eta(\jmath)}}\right)^{n}\right)^{ℶ_\jmath}}\end{pmatrix}$$

where $ℶ=(\boldsymbol{I}_1,ℶ_2,\dots.,\boldsymbol{J}_d)^T$ are the weights of $\mathbb{S}_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}ℶ_\jmath=1$ and gth biggest weighted value is $\mathbb{S}_{\eta(\jmath)}\left(\mathbb{S}_{\eta(\jmath)}=\eta ℶ_\jmath\mathbb{S}_{\eta(\jmath)}\right)$. Also the associated weighted vector $\omega_\jmath$ with $\omega_\jmath\in[0,1]$ with $\sum_{\jmath=1}^{d}\omega_\jmath=1$

5 MAGDM Based on N-ROTFNS

This section established a framework for solving MAGDM issues under n-ROtF information.

Let $\left\{\partial^{a}{}_{1},\partial^{a}{}_{2},\dots\dots.,\partial^{a}{}_{p}\right\}$ be a set of p alternatives and let $\{N_1,N_2,\dots.,N_d\}$ be a set of attributes/criteria. The weight vector is denoted by $ℶ=(ℶ_1,ℶ_2,\dots.,ℶ_d)$ where $J_t\in[0,1]$ and $\sum_{t=1}^{d}ℶ_t=1$. Consider the matrix, $\mathrm{C}=[\mathbb{S}_{kt}]_{p\times d}=\left[\left(R_{\mathbb{S}_{kt}},S_{\mathbb{S}_{kt}},T_{\mathbb{S}_{kt}}\right)\right]_{p\times d}$ represents the n-ROtF information.

$$\begin{array}{c} \\ \eth^a{}_1 \\ \eth^a{}_2 \\ \cdots \\ \eth^a{}_p \end{array} \begin{array}{c} \begin{array}{cccc} N_1 & N_2 & \cdots & N_d \end{array} \\ \begin{bmatrix} (R_{\mathbb{S}_{11}}, S_{\mathbb{S}_{11}}, T_{\mathbb{S}_{11}}) & (R_{\mathbb{S}_{12}}, S_{\mathbb{S}_{12}}, T_{\mathbb{S}_{12}}) & \cdots & (R_{\mathbb{S}_{1d}}, S_{\mathbb{S}_{1d}}, T_{\mathbb{S}_{1d}}) \\ (R_{\mathbb{S}_{21}}, S_{\mathbb{S}_{21}}, T_{\mathbb{S}_{21}}) & (R_{\mathbb{S}_{22}}, S_{\mathbb{S}_{22}}, T_{\mathbb{S}_{22}}) & \cdots & (R_{\mathbb{S}_{2d}}, S_{\mathbb{S}_{2d}}, T_{\mathbb{S}_{2d}}) \\ \cdots & \cdots & \cdots & \cdots \\ (R_{\mathbb{S}_{p1}}, S_{\mathbb{S}_{p1}}, T_{\mathbb{S}_{p1}}) & (R_{\mathbb{S}_{p2}}, S_{\mathbb{S}_{p2}}, T_{\mathbb{S}_{p2}}) & \cdots & (R_{\mathbb{S}_{pd}}, S_{\mathbb{S}_{pd}}, T_{\mathbb{S}_{pd}}) \end{bmatrix} \end{array}$$

Key steps are described as:

Step 1: Establishing the n-ROtF decision matrices

$$C = [\mathbb{S}_{kt}]_{p\times d} = \left[\left(R_{\mathbb{S}_{kt}}, S_{\mathbb{S}_{kt}}, T_{\mathbb{S}_{kt}}\right)\right]_{p\times d}$$
$$(k = 1,2, \dots, p; t = 1,2, \dots, d)$$

Step 2: Exploit the n-ROtFWA or n-ROtFWG operator to achieve the overall matrix

$$n - \text{ROtFW } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \left(\begin{array}{c} \sqrt[n]{1 - \Pi_{j=1}^{d} \left(1 - \left(R_{\mathbb{S}_j}\right)^n\right)^{\beth_j}}, \Pi_{j=1}^{d} \left(S_{\mathbb{S}_j}\right)^{\Xi_j}, \\ , \Pi_{j=1}^{d} \left(T_{\mathbb{S}_j}\right)^{\beth_j} \end{array} \right)$$

Step 3: After that, we compute the scores $s(\partial^a{}_k)(k = 1,2, \dots, p)$ and the accuracy degrees $s(\partial^a{}_k)(k = 1,2, \dots, p)$ of all the overall values $\partial^a{}_k(k = 1,2, \dots, p)$.

Step 4: According to Definition 15, Rank the alternatives $\partial^a{}_k(k = 1,2, \dots, p)$ and afterward select the best one.

6 Numerical Example

To validate our established algorithm we consider the case of selecting a optimal way to handle food security in Pakistan.

6.1 Case Study.

Food security refers to the availability access, and utilization of food by individuals and households. In Pakistan, there are several factors that can affect food security, including poverty, natural disasters, and conflict (Zhou et al., 2019).

There are several criteria that can be used to measure food security in Pakistan.

N_1.Access to food: This refers to the availability of food within a reasonable distance of the household, as well as the ability of the household to purchase food. N_2.Food utilization: This refers to the ability of the household to prepare and consume sufficient, safe, and nutritious food.

N_3.Food stability: This refers to the ability of the household to access food consistently over time.

N_4.Food diversity: This refers to the variety of different types of food consumed by the household.

There are several alternatives that can be implemented to improve food security in Pakistan. These include

$\partial^{a}{}_{1}$. Increasing agricultural productivity: This can be done through the use of modern technology, improved seeds, and better irrigation systems.

$\partial^{a}{}_{2}$. Strengthening the food distribution system: This can be done through the development of more efficient supply chains, as well as the expansion of social protection programs such as food assistance and cash transfers.

$\partial^{a}{}_{3}$. Promoting food-based livelihoods: This can be done through the development of small-scale agriculture and agribusiness enterprises, which can help to create jobs and increase household incomes.

$\partial^{a}{}_{4}$. Improving food safety and nutrition: This can be done through the implementation of food safety regulations, as well as nutrition education programs. $\partial^{a}{}_{5}$. Enhancing resilience to shocks: This can be done through the development of disaster risk reduction strategies, such as the construction of early warning systems and the implementation of risk-reducing practices in agriculture.

The weight vector is $\varphi = (0.110, 0.204, 0.355, 0.331)^{T}$.

The estimation values of the alternatives regarding each criterion provided by the specialists are developed by $n - ROtFN$ s as revealed in the n-ROtF decision matrix given in Table-1. To solve the MCDM issue by developed operators, the following calculations are achieved:

Table 1: Normalized Collective Data of Experts

	N_1	N_2	N_3	N_4
$\partial^{a}{}_{1}$	(0.4,0.2,0.8)	(0.2,0.3,0.9)	(0.1,0.1,0.9)	(0.1,0.1,0.9)
$\partial^{a}{}_{2}$	(0.9,0.1,0.1)	(0.8,0.2,0.4)	(0.8,0.2,0.5)	(0.9,0.3,0.2)
$\partial^{a}{}_{3}$	(0.6,0.2,0.6)	(0.5,0.2,0.8)	(0.1,0.1,0.9)	(0.9,0.3,0.2)
$\partial^{a}{}_{4}$	(0.2,0.3,0.9)	(0.6,0.2,0.6)	(0.2,0.6,0.6)	(0.2,0.4,0.8)
$\partial^{a}{}_{5}$	(0.8,0.2,0.4)	(0.5,0.2,0.8)	(0.1,0.9,0.1)	(0.9,0.1,0.1)

Step 2: Exploit the established aggregation operator to achieve the n-ROtFN $\mathbb{S}_{k} (k = 1,2, \dots, d)$ for the alternatives $\partial^{a}{}_{k}$ with n $= 2$

$$n - \text{ROtFWA } A(\mathbb{S}_1, \mathbb{S}_2, \dots, \mathbb{S}_d) = \begin{pmatrix} \sqrt[n]{1 - \Pi_{j=1}^{d} \left(1 - \left(R_{\mathbb{S}_j}\right)^{n}\right)^{\beth_j}}, \Pi_{j=1}^{d} \left(S_{\mathbb{S}_j}\right)^{\beth_j} \\ , \Pi_{j=1}^{d} \left(T_{\mathbb{S}_j}\right)^{\beth_j} \end{pmatrix}$$

Table 2: Weighted Normalized Matrix

	N
$\partial^a{}_1$	(0.6950,0.1350,0.8884)
$\partial^a{}_2$	(0.9919,0.2119,0.2955)
$\partial^a{}_3$	(0.9846,0.1788,0.5108)
$\partial^a{}_4$	(0.7937,0.3885,0.6900)
$\partial^a{}_5$	(0.9855,0.2712,0.1780)

Step 3: After that, we compute the scores

Table 3: Score

	N
$\partial^a{}_1$	0.5579
$\partial^a{}_2$	0.9496
$\partial^a{}_3$	0.8913
$\partial^a{}_4$	0.6670
$\partial^a{}_5$	0.9544

Step 4: According to Definition 15, Rank the alternatives $\partial^a{}_k(k = 1,2, \ldots ,5)$ and afterward select the best one.

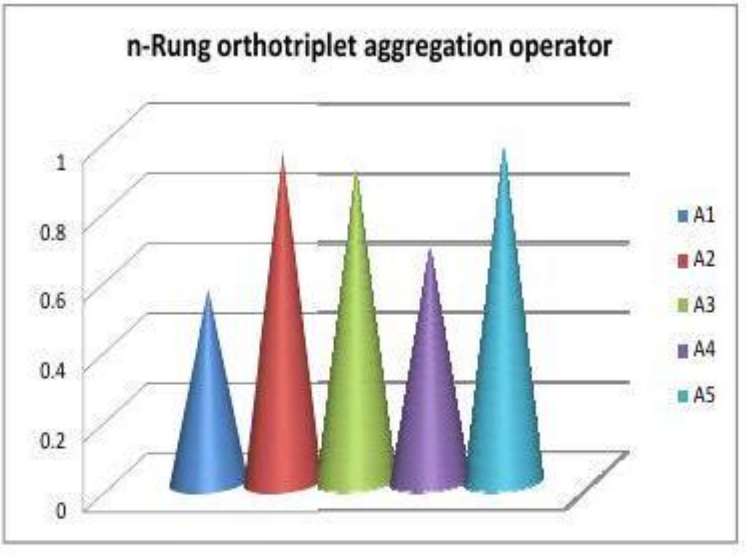

Figure 1: n-rung orthotriplet aggregation operator.

We can conclude from this above computational process that $\partial^a{}_5$ is the best alternative. Therefore it is highly recommended to enhance resilience to shocks and this can be done through the development of disaster risk reduction strategies, such as the construction of early warning systems and the implementation of risk-reducing practices in agriculture.

6.2 Impact of parametern.

The method shown that the picture fuzzy set and spherical fuzzy set are bound in limited domain but in case of n-ROtF information decision makers are flexible by adapting the parameter n (Mahmood et al., 2019). Picture and spherical fuzzy failed to examine the information when values are (0.8,0.6,0.5). On the other hand, the n-ROtF set have an adjustable parameter which can easily manage such types of information. n-ROtF operators are more powerful than the existing operators. Thus, there is a broader range to manage the data by parameter n than others and hence the corresponding method is more adaptable to solve decision making problems.

Table 3: Comparison matrix

	$\partial^{a}{}_{1}$	$\partial^{a}{}_{2}$	$\partial^{a}{}_{3}$	$\partial^{a}{}_{4}$	$\partial^{a}{}_{5}$	Ranking
$n = 2$	0.5580	0.9496	0.8913	0.6670	0.9544	$\partial^{a}{}_{5} \succ \partial^{a}{}_{2} \succ \partial^{a}{}_{3}{}^{3} \succ \partial^{a}{}_{4} \succ \partial^{a}{}_{1}$
$n = 3$	0.6022	0.9514	0.8946	0.7020	0.9575	$\partial^{a}{}_{5} \succ \partial^{a}{}_{2} \succ \partial^{a}{}_{3} \succ \partial^{a}{}_{4} \succ \partial^{a}{}_{1}$
$n = 4$	0.6285	0.9523	0.8963	0.7215	0.9591	$\partial^{a}{}_{5} \succ \partial^{a}{}_{2} \succ \partial^{a}{}_{3} \succ \partial^{a}{}_{4} \succ \partial^{a}{}_{1}$
$n = 5$	0.6460	0.9528	0.8973	0.7340	0.9601	$\partial^{a}{}_{5} \succ \partial^{a}{}_{2} \succ \partial^{a}{}_{3} \succ \partial^{a}{}_{4} \succ \partial^{a}{}_{1}$
$n = 6$	0.6582	0.9531	0.8980	0.7425	0.9607	$\partial^{a}{}_{5} \succ \partial^{a}{}_{2} \succ \partial^{a}{}_{3} \succ \partial^{a}{}_{4\,4} \succ \partial^{a}{}_{1}$

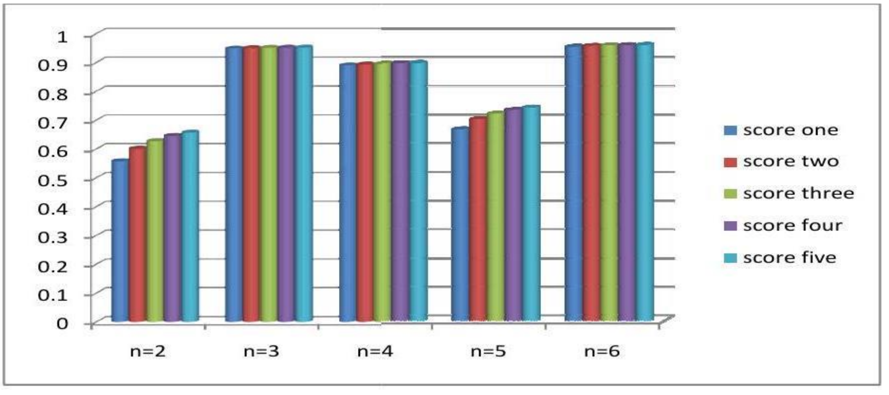

Figure 2: Impact of parameter n.

7 Conclusion

In this manuscript, we have proposed a novel hybrid concept of n-rung ortho triplet fuzzy set to tackle the random vagueness, which is an enhanced version of fuzzy set, IFS, PyFS in decision making real life problems. Picture fuzzy set and spherical fuzzy set are special cases of n-rung ortho triplet fuzzy set. It considers positive, negative and neutral membership grades so in this case there are more options for experts to take a decision whereas in case of pythagorean fuzzy set there are only positive and negative membership grades. We established a multi attribute decision-making approach (MADMap) based on the n-rung ortho triplet fuzzy information In addition using proposed technique, we developed an algorithm to tackle MADM problems. We developed a method to select an ideal alternative from certain alternatives for food security. The aim of this manuscript is to present n-ROtF- aggregation method based on n-ROtF averaging and geometric aggregation operator. The n-ROtFN in managing with uncertain data to establish decision-making algorithm and select an appropriate alternative to tackle food security issue. The main benefit of established algorithm is that it takes the neutral information alongwith positive and negative membership degrees into account which give more details without any loss of information. The established algorithm has been signified with a food security issue to show the validity and effectiveness of our established technique under n-ROtF information.

8 References

Akhmouch, A., Clavreul, D., Hendry, S., Megdal, S. B., Nickum, J. E., Nunes-Correia, F., & Ross, A. (2020). Introduction: Introducing the OECD Principles on Water Governance. In A. Akhmouch. D. Clavreul. S. Hendry. S. Megdal. J. Nickum. F. Nunes-Correia. & A. Ross. (Eds.), *OECD Principles on Water Governance* (pp. 5–12). Routledge. https://doi.org/10.4324/9780429448058

Akram, M., Ali, G., & Alcantud, J. C. R. (2023). A novel group decision-making framework under Pythagorean fuzzy N-soft expert knowledge. *Engineering Applications of Artificial Intelligence*, *120*, Article e105879. https://doi.org/10.1016/j.engappai.2023.105879

Alamoodi, A. H., Albahri, O. S., Zaidan, A., AlSattar, H. A., Ahmed, M. A., Pamucar, D., Zaidan, B., Albahri, A. S., & Mahmoud, M. S. (2022). New extension of fuzzy-weighted zero-inconsistency and fuzzy decision by opinion score method based on cubic pythagorean fuzzy environment: a benchmarking case study of sign language recognition systems. *International Journal of Fuzzy Systems*, *24*(4), 1909–1926. https://doi.org/10.1007/s40815-021-01246-z

Ali, Z., Mahmood, T., & Yang, M.-S. (2020). Complex T-spherical fuzzy aggregation operators with application to multi-attribute decision making. *Symmetry*, *12*(8), Article e1311. https://doi.org/10.3390/sym12081311

Arya, V., & Kumar, S. (2021). Extended TODIM method based on VIKOR for q-rung orthopair fuzzy information measures and their application in MAGDM problem of medical consumption products. *International Journal of Intelligent Systems*, *36*(11), 6837–6870. https://doi.org/10.3390/sym12081311

Ashraf, S., Abdullah, S., Mahmood, T., Ghani, F., & Mahmood, T. (2019). Spherical fuzzy sets and their applications in multi-attribute decision making problems. *Journal of Intelligent & Fuzzy Systems*, *36*(3), 2829–2844.

Ayyildiz, E., & Taskin, A. (2022). A novel spherical fuzzy AHP-VIKOR methodology to determine serving petrol station selection during COVID-19 lockdown: A pilot study for İstanbul. *Socio-Economic Planning Sciences*, *83*, Article e101345. https://doi.org/10.1016/j.seps.2022.101345

Batool, B., Abdullah, S., Ashraf, S., & Ahmad, M. (2022a). Pythagorean probabilistic hesitant fuzzy aggregation operators and their application in decision-making. *Kybernetes*, *51*(4), 1626–1652. https://doi.org/10.1108/K-11-2020-0747

Batool, B., Abosuliman, S. S., Abdullah, S., & Ashraf, S. (2022b). EDAS method for decision support modeling under the Pythagorean probabilistic hesitant fuzzy aggregation information. *Journal of Ambient Intelligence and Humanized Computing*, 1–14. https://doi.org/10.1007/s12652-021-03181-1

Din, M. S. U., Mubeen, M., Hussain, S., Ahmad, A., Hussain, N., Ali, M. A., Sabagh, A. E., Elsabagh, M., Shah, G. M., Qaisrani, S. A., Tahir, M., Javeed, H. M. R., Anwar-Ul-Haq, M., Ali, M., & Nasim, W. (2021). World Nations priorities on climate change and food security. In *Springer eBooks* (pp. 365–384). https://doi.org/10.1007/978-3-030-79408-8_22

Guleria, A., & Bajaj, R. K. (2020). A novel probabilistic distance measure for picture fuzzy sets with its application in classification problems. *Hacettepe Journal of Mathematics and Statistics*, *49*(6), 2134-2153. https://doi.org/10.15672/hujms.677920

He, X., & Wu, Y. (2019). Global research trends of intuitionistic fuzzy set: A bibliometric analysis. *Journal of Intelligent Systems*, *28*(4), 621–631. https://doi.org/10.1515/jisys-2017-0240

Jin, H., Jah Rizvi, S. K., Mahmood, T., Jan, N., Ullah, K., & Saleem, S. (2020). An intelligent and robust framework towards anomaly detection, medical diagnosis, and shortest path problems based on interval-valued T-spherical fuzzy information. *Mathematical Problems in Engineering*, *2020*, 1–23. https://doi.org/10.1155/2020/9656909

Karamaz, F., & Karaaslan, F. (2021). Hesitant fuzzy parameterized soft sets and their applications in decision making. *Journal of Ambient Intelligence and Humanized Computing*, *12*, 1869–1878. https://doi.org/10.1007/s12652-020-02258-7

Khan, M. S. A., Abdullah, S., Ali, A., Siddiqui, N., & Amin, F. (2017). Pythagorean hesitant fuzzy sets and their application to group decision making with incomplete weight information. *Journal of Intelligent & Fuzzy Systems*, *33*(6), 3971–3985. https://doi.org/10.3233/JIFS-17811

Mardani, A., Hooker, R. E., Ozkul, S., Yifan, S., Nilashi, M., Sabzi, H. Z., & Fei, G. C. (2019). Application of decision making and fuzzy sets theory to evaluate the healthcare and medical problems: a review of three decades of research with recent developments. *Expert Systems with Applications*, *137*, 202–231. https://doi.org/10.1016/j.eswa.2019.07.002

Mahmood, T., Ullah, K., Khan, Q., & Jan, N. (2019). An approach toward decision-making and medical diagnosis problems using the concept of spherical fuzzy sets. *Neural Computing and Applications*, *31*, 7041–7053. https://doi.org/10.1007/s00521-018-3521-2

Peleg, M., & Tu, S. (2006). Section 5: Decision support, knowledge representation and management: Decision support, knowledge representation and management in medicine. *Yearbook of Medical Informatics*, *15*(01), 72–80. https://doi.org/10.1055/s-0038-1638482

Rehman, A., Ma, H., Ozturk, I., & Ulucak, R. (2022). Sustainable development and pollution: The effects of CO 2 emission on population growth, food production, economic development, and energy consumption in Pakistan. *Environmental Science and Pollution Research*, *29*, 1–12. https://doi.org/10.1007/s11356-021-16998-2

Sakr, H. H., Muse, A. H., Mohamed, M. S., & Ateya, S. F. (2023). Applications on bipolar vague soft sets. *Journal of Mathematics*, *2023*, 1–25.

Smarandache, F. (2019). Neutrosophic set is a generalization of intuitionistic fuzzy set, inconsistent intuitionistic fuzzy set (picture fuzzy set, ternary fuzzy set), pythagorean fuzzy set, spherical fuzzy set, and q-rung orthopair fuzzy set, while neutrosophication is a generalization of regret theory, grey system theory, and three-ways decision (revisited). *Journal of New Theory*, *20*, 1–31.

Xu, Z., & Zhou, W. (2017). Consensus building with a group of decision makers under the hesitant probabilistic fuzzy environment. *Fuzzy Optimization and Decision Making*, *16*, 481–503. https://doi.org/10.1007/s10700-016-9257-5

Yager, R. R. (2016). Generalized orthopair fuzzy sets. *IEEE transactions on fuzzy systems*, *25*(5), 1222-1230. https://doi.org/10.1109/TFUZZ.2016.2604005

Yager, R. R., & Abbasov, A. M. (2013). Pythagorean membership grades, complex numbers, and decision making. *International Journal of Intelligent Systems*, *28*(5), 436–452. https://doi.org/10.1002/int.21584

Zhou, D., Shah, T., Ali, S., Ahmad, W., Din, I. U., & Ilyas, A. (2019). Factors affecting household food security in rural northern hinterland of Pakistan. *Journal of the Saudi Society of Agricultural Sciences*, *18*(2), 201–210. https://doi.org/10.1016/j.jssas.2017.05.003